Retaining Walls for Non-Geotechnical Engineers

Course No: G02-013

Credit: 2 PDH

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RETAINING WALLS FOR NON-GEOTECHNICAL ENGINEERS
By Richard P. Weber

Introduction

Retaining walls are structures that support backfill and allow for a change of grade. For instance a retaining wall can be used to retain fill along a slope or it can be used to support a cut into a slope.

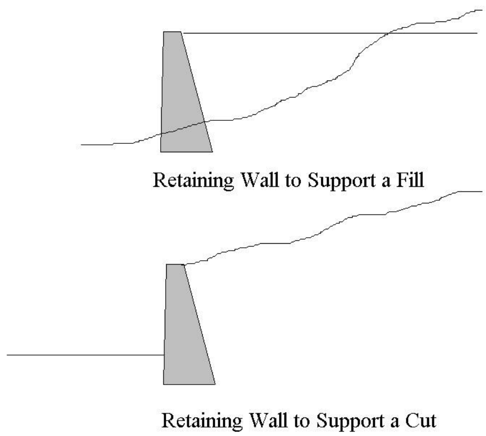

Figure 1 – Example of Retaining Walls

Retaining wall structures can be gravity type structures, semi-gravity type structures, cantilever type structures, and counterfort type structures. Walls might be constructed from materials such as fieldstone, reinforced concrete, gabions, reinforced earth, steel and timber. Each of these walls must be designed to resist the external forces applied to the wall from earth pressure, surcharge load, water, earthquake etc.

The objective of this course is to familiarize primarily the non-geotechnical engineer with methods for calculating the active earth pressure force against a retaining wall and for assessing its stability with respect to sliding, overturning and bearing capacity.

This course is not meant to be exhaustive nor does it discuss the structural design of a retaining wall. There are many textbooks and publications that treat detailed loading conditions in depth including:

- Foundations and Earth Structures, NAVFAC, Design Manual 7.2
- Retaining and Flood Walls, Technical Engineering and Design Guides As Adapted from The US Army Corps Of Engineers, No. 4, ASCE
- Standard Specifications for Highway Bridges, AASHTO

At the end of this course you will have learned:

- Calculating the lateral earth pressure force.
- Calculating factors of safety for overturning, sliding and bearing capacity.

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The reader should already be familiar with methods for calculating the earth pressure. If not, textbooks or the companion course Lateral Earth Pressure for Non-Geotechnical Engineers will be helpful.

Lateral Earth Pressure

There are three categories of lateral earth pressure:

- At Rest Pressure
- Passive Pressure
- Active Pressure

This course only discusses the active earth pressure because it is the active pressure that produces the destabilizing earth force behind retaining walls. Although passive pressures might develop along the toe of the wall and provide resistance it is commonly ignored and therefore not discussed in this course. This course also does not discuss other kinds of lateral forces, such as those resulting from surcharge, earthquake, etc., which also produce additional destabilizing forces.

Since soil backfill is typically granular material such as sand, silty sand, sand with gravel, this course assumes that the backfill material against the wall is coarse-grained, non-cohesive material. Thus, cohesive soil such as clay is not discussed.

The lateral earth pressure is equal to vertical effective overburden pressure times the appropriate earth pressure coefficient. There are published relationships, tables and charts for calculating or selecting the appropriate earth pressure coefficient.

When calculating the lateral earth pressure force there are two methods that are widely used:

- Rankine Earth Pressure
- Coulomb Earth Pressure

The Rankine method assumes:

- There is no adhesion or friction between the wall and soil
- Lateral pressure is limited to vertical walls
- Failure (in the backfill) occurs as a sliding wedge along an assumed failure plane defined by the friction angle of the soil backfill (ϕ).
- Lateral pressure varies linearly with depth and the resultant pressure is located one-third of the height (H) above the base of the wall.
- The resultant force is parallel to the backfill surface.

The Coulomb method is similar to Rankine except that:

- There is friction between the wall and soil and takes this into account by using a soil-wall friction angle of δ. Note that δ ranges from $\phi/2$ to $2\phi/3$ and $\delta = 2\phi/3$ is commonly used.
- Lateral pressure is <u>not</u> limited to vertical walls
- The resultant force is not necessarily parallel to the backfill surface because of the soil-wall friction value (δ).

It is important to note that the <u>full active earth pressure</u> condition will only develop if the wall is allowed to <u>move</u> a sufficient distance. The lateral outward movement required to develop the full active pressure condition ranges from:

- Granular soil: 0.001H to 0.004H
- Cohesive soil: 0.01H to 0.04H

Where H is the height of the wall.

The <u>Rankine active earth pressure coefficient</u> for the specific condition of a <u>horizontal</u> backfill surface is calculated as follows:

- $Ka = (1 - \sin(\phi)) / (1 + \sin(\phi))$ (1.0)

The expression is modified if the backfill surface were sloped.

The <u>Coulomb active earth pressure coefficient</u> is a more complicated expression that depends on the angle of the back of the wall, the soil-wall friction and the angle of backfill slope. Although the expression is not shown, these values are readily obtained in textbook tables or by programmed computers and calculators. The Table below shows some examples of the Coulomb active earth pressure coefficient for the specific case of a wall with a <u>back of wall angle (β)</u> of 80 degrees and a <u>horizontal</u> backfill surface. In this Table, the soil-wall friction value (δ) has been taken as $(2/3)\phi$.

Table 1 - Coulomb Active Pressure Coefficient
[Note: $\delta = (2/3)\phi$]

ϕ (deg)	β = 80 deg
28	.4007
30	.3769
32	.3545

If water pressure were allowed to build up behind a retaining wall, then the total pressure and the resulting total force along the back of the wall is increased considerably. Therefore, it is common for walls to be designed with adequate drainage to prevent water from accumulating behind the wall and producing large additional forces. Thus, weepholes, lateral drains or blanket drains along with granular soil (freely draining backfill) are commonly used behind retaining walls.

Calculating the Total Active Earth Pressure Force

The total lateral force is the area of the pressure diagram acting on the wall surface. The examples in this course assume drained conditions and a homogeneous granular soil backfill behind the wall, which results in a simple triangular distribution. Although this is a common case, the pressure diagram can become more complicated depending upon actual soil conditions that might have different values.

With the Coulomb method, the active force acts directly on the wall and friction develops between the soil and wall. With the Rankine method however, wall friction is ignored and the active force acts directly on a vertical face extending through the heel of the wall. If the back of the wall were vertical, then the force acts on the wall. On the other hand, if the back of the wall were sloping, then the force acts on the vertical soil plane as illustrated in Figure 2.

In the example shown later in this course, the area of the earth pressure diagram is the earth pressure at the bottom of the wall ($Ka\gamma H$) times the height of the wall (H) times one-half (1/2) since the pressure distribution increases linearly with depth creating a triangular shape. Thus the total active earth pressure force (Pa) acting along the back of the wall is the area of the pressure diagram expressed as:

- $Pa = \frac{1}{2}\ Ka\ \gamma\ H^2$ (2.0)

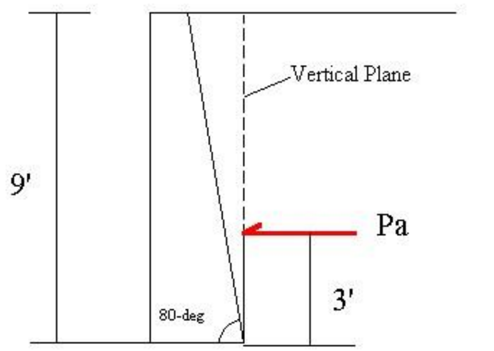
The total force acts along the back of the wall at a height of H/3 from the base of the wall. So far we have not stated whether this is the Rankine or Coulomb Case. The calculation for the active earth pressure force (Pa) is the same provided that the appropriate earth pressure coefficient (Ka) is used. Selecting whether the Rankine method or Coulomb method will be used is usually a matter of choice or convention.

The example shown in Figure 2 relates specifically to a wall supporting a horizontal backfill. Thus the active earth pressure coefficient (Ka) can be derived directly from Expression (1.0) or Table 1. For the case of a sloping backfill and other wall geometries, the reader should refer to the published references.

This example assumes that a 9-foot high gravity type retaining structure supports soil backfill having a total unit weight of 125 pcf. Groundwater is well below the structure and the backfill material is freely draining. The backfill soil has an angle of internal friction (ϕ) of 32 degrees and the backfill surface behind the wall is horizontal. Both the Rankine and Coulomb earth pressure force is shown.

Note that the location and direction of the active forces follows the assumptions stated above for the Rankine and Coulomb Theory. Although the back of the wall has an angle of 80 degrees, The Rankine force acts along a vertical plane beginning at the heel of the wall while the Coulomb force acts directly along the back of the wall. Since the Rankine Theory assumes that there is no soil – wall friction, the force (Pa) is parallel to the backfill surface. On the other hand, since the Coulomb Theory takes the soil – wall friction into consideration, the force (Pa) acts at an angle of δ from the perpendicular to the wall. This results in both a vertical and horizontal component of the force (Pa). The Rankine method will also produce a vertical and horizontal component of the force (Pa) if the backfill surface has a slope.

In each case, the resultant force Pa acts at a height of H/3 from the base of the wall where H is the height of the wall for the simple case illustrated herein. If the pressure diagram were more complicated due to differing soil conditions for instance, then the location of the force (Pa) will change. In all cases however, the resultant of the force (Pa) is located at the centroid of the combined mass area.

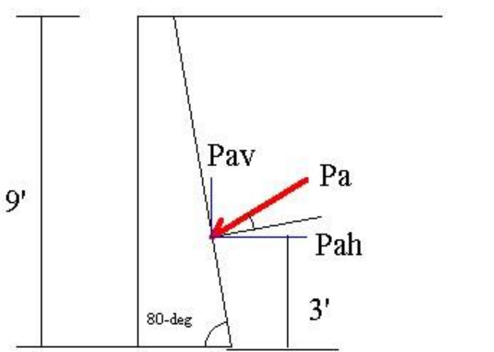


Rankine Active Earth Pressure Force

$Ka = (1 - \sin(\phi)) / (1 + \sin(\phi)) = 0.307$

$Pa = ½\ Ka\ \gamma\ H^2 = (0.5)(0.307)(125)(9^2)$

Pa = 1554.2 pounds



Coulomb Active Earth Pressure Force

Ka = 0.3545 from Table 1 for conditions stated

$Pa = ½\ Ka\ \gamma\ H^2 = (0.5)(0.354)(125)(9^2)$

Pa = 1792.1 pounds

Calculate horizontal and vertical components of Pa where Pa acts 31.3 deg from the horizontal.

Pa_h = Pa cos (31.3) = 1531.3 pounds

Pa_v = Pa sin (31.3) = 931.0 pounds

Figure 2 - Calculation of Earth Pressure Force for a Homogeneous Cohesionless Backfill

Other Forces Acting on the Wall

Aside from the earth pressure force acting on wall, other forces might also act on the wall. Although these forces are not discussed in this course, they might include:

- Surcharge load
- Earthquake load
- Water Pressure

These additional forces would be superimposed onto the earth pressure force to yield the total lateral force.

Factors of Safety

Retaining wall design is an iterative process. An initial geometry is assigned to the wall and the appropriate forces are calculated. The actual forces are then checked using appropriate factors of safety and the geometry is revised until satisfactory factors of safety are reached. There are common dimensions that are available that can be used as a first cut.

Proportioning Walls

In order to achieve stability, retaining walls are usually proportioned so that the width of the base (B) is equal to approximately 0.5 to 0.7 times the height of the wall (H). This is illustrated in Figure 3. Thus, a 9-foot high wall would have a base approximately 4.5 feet to 6.3 feet wide which provides a convenient starting point.

Sliding

A retaining structure has a tendency to move away from the backfill surface because of the horizontal driving forces resulting from the soil backfill and other forces such as surcharge. Generally, the wall resists sliding by the frictional resistance developed between the foundation of the wall and foundation soil. Although other horizontal forces act opposite to the driving force such as passive soil pressure in the fill in front of the wall, it is often ignored.

The factor of safety with respect to sliding equals the resisting force divided by the driving force and is shown in Expression (3.0). A minimum factor of safety of 1.5 is desirable to resist sliding assuming that passive resistance from any fill in front of the wall is ignored. This is a common assumption and avoids relying on the presence of soil in front of the wall for additional resistance.

$$FS_s = \Sigma V \tan(k\phi_1) / Pa_h \qquad (3.0)$$

ΣV is the total vertical force, Pa_h is the horizontal active earth pressure force and $\tan(k\phi_1)$ is the coefficient of friction between the base of the wall and the soil. The factor "k" ranges from ½ to 2/3 and ϕ_1 is the friction angle of the foundation soil. Friction factors between dissimilar materials can also be found in publications such as NAVFAC Design Manual 7.2. Expression (3.0) assumes that the soil below the wall is a cohesionless material such as sand without any cohesive strength. Therefore there is no additional resistance due to cohesion.

Overturning

A retaining structure also has a tendency to rotate outward around the toe of the wall. The moment resulting from the earth pressure force (as well as other lateral forces such as surcharge) must be resisted by the moments resulting from the vertical forces produced by the wall including any vertical component (Pa_v) of the earth pressure force. Thus, the factor of safety with respect to overturning is the resisting moment divided by the overturning moment as shown in Expression (4.0). A minimum factor of safety of 2 to 3 is desirable to resist overturning.

$$FS_o = \Sigma Mr / \Sigma Mo \qquad (4.0)$$

Where ΣMr is the sum of the resisting moments around the toe of the wall and ΣMo is the sum of the overturning moments around the toe of the wall.

Bearing Capacity

As with any structure, the bearing capacity of the soil must be adequate to safely support the structure. The ultimate bearing capacity of the foundation soil (qu) is calculated using theoretical bearing capacity methods presented in textbooks and other published resources. The calculation of bearing capacity is not presented in this course.

The resultant of all forces acting along the base of the wall from earth pressure and the weight of the wall result in a non-uniform pressure below the base of the wall with the greatest pressure below the toe of the base and the least pressure below the heel of the base.

The maximum and minimum pressure below the base of the wall (B) is:

$$q_{max} = (\Sigma V / B)(1 + 6e / B) \qquad (5.0)$$

$$q_{min} = (\Sigma V / B)(1 - 6e / B) \qquad (6.0)$$

$$\text{Where } e = \text{eccentricity};\ e = (B / 2) - (\Sigma Mr - \Sigma Mo) / \Sigma V \qquad (7.0)$$

The factor of safety with respect to bearing capacity is shown in Expression (8.0). Generally, a factor of safety of 3 is required.

$$FS_{bc} = q_u / q_{max} \qquad (8.0)$$

Eccentricity is an important consideration when proportioning the walls. Consider the eccentricity (e) in relationship to the minimum pressure (q_{min}). Substituting for (e) in Expression (6.0):

$$\text{If } e = B / 6 \text{ then } q_{min} = (\Sigma V / B)\ (1 - 6e / B) = 0 \qquad (9.0)$$

$$\text{If } e < B / 6 \text{ then } q_{min} = (\Sigma V / B)\ (1 - 6e / B) > 0 \qquad (10.0)$$

$$\text{If } e > B / 6 \text{ then } q_{min} = (\Sigma V / B)\ (1 - 6e / B) < 0 \qquad (11.0)$$

Expressions (9.0) and (10.0) give acceptable results since the pressure at the heel is zero or greater (positive). Thus the entire base lies in contact with the soil. If Expression (11.0) were true, then the pressure at the heel is negative indicating the heel of the base is tending toward lifting off the soil, which is unacceptable. If this condition occurs, then the wall must be re-proportioned.

Other Considerations

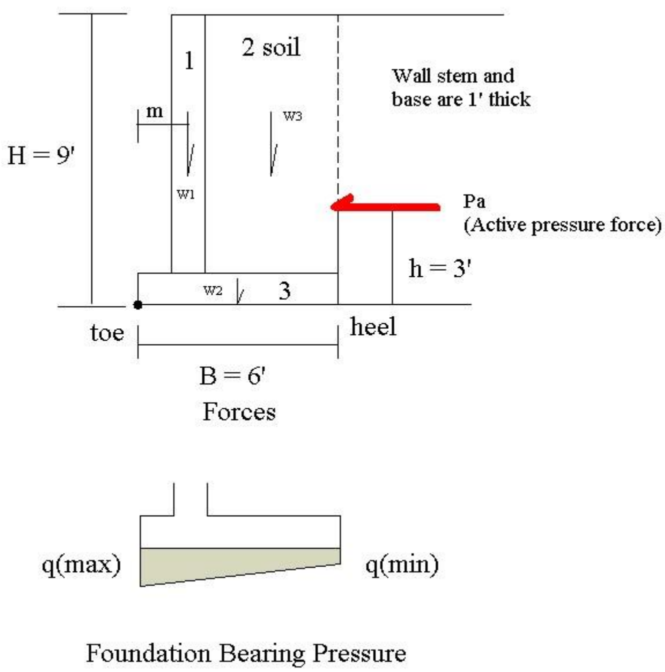
Before a wall design is complete, the settlement of the wall and the global stability of the entire mass on which the wall is supported must be checked. Settlement must lie within tolerable ranges and global stability such as from slope stability calculations must be adequate. These calculations however, are beyond the scope of this course.

Example

The following example illustrates the discussion presented in this course.

Using the Rankine method of analysis, calculate the factors of safety with respect to sliding, overturning and bearing capacity. Use the values presented in the following Table and refer to Figure 3. It is inferred that all calculations relate to a unit length of wall.

Friction angle of soil backfill (ϕ)	32 degrees
Soil Backfill Unit Weight (γ)	125 pcf
Friction angle of the foundation soil (ϕ_1)	33 degrees
Rankine active pressure coefficient (Ka)	0.307
Concrete Unit Weight (γ_c)	150 pcf
Dimensions of the concrete wall section 1	1-ft by 8-ft
Dimensions of the soil backfill section 2	4-ft by 8-ft
Dimensions of the concrete wall section 3	6-ft by 1-ft



Note:

Since Pa is horizontal, there is no vertical component of the force. If the backfill surface were sloping then Pa would slope at an angle parallel to the backfill slope. In this case there would be both a vertical and horizontal component of Pa. The lateral thrust would be the horizontal component and the vertical component would be an additional vertical force included in ΣV.

Figure 3 - Example

<u>Solution</u>

Calculate the values shown in the following Table. The dimensions for "Area" relate to each of the three sections identified in Figure 3. The unit weight (γ) is provided for the concrete wall and soil backfill over the base of the wall. W is the weight of each section and it acts at the centroid of the mass area as shown in Figure 3. The value "m" is the moment arm measured from the toe to the location of the individual W values. M is the resisting moment for each of the individual areas.

Section	Area (sf)	γ (pcf)	W (lbs)	m (ft)	M (ft-lb)
1	1 x 8	150	1200	1.5	1800
2	4 x 8	125	4000	4	16000
3	6 x 1	150	900	3	2700
			ΣV = **6100**		ΣMr = **20500**

Pa = ½ Ka γ H^2 = (0.5) (0.307) (125) (81) = <u>**1554.2 lbs**</u>

ΣMo = Pa (h) = (1554.2) (3) = <u>**4662.6 ft-lbs**</u>

Overturning: $Fs_o = \Sigma Mr / \Sigma Mo = 20500 / 4662.6 = \mathbf{4.4} > 2$ **OK**

Sliding: $FS_s = \Sigma V \tan(k\phi_1) / Pa_h = (6100) \tan(22) / 1554.2 = \mathbf{1.58} > 1.5$ **OK**
Where $k = 2/3$

Bearing Capacity:

Assume that the ultimate bearing capacity of the foundation soil is 5000 psf.

$e = (B / 2) - (\Sigma Mr - \Sigma Mo) / \Sigma V = (6 / 2) - (20500 - 4662.6) / 6100 = \mathbf{0.4}$ (i.e. $e < B / 6$)

$q_{max} = (\Sigma V / B)(1 + 6e / B) = (6100 / 6)(1 + 2.4 / 6) = (1016.6)(1.4) = \underline{\mathbf{1423.4\ psf}}$

$q_{min} = (\Sigma V / B)(1 - 6e / B) = (6100 / 6)(1 - 2.4 / 6) = (1016.6)(.6) = \underline{\mathbf{610\ psf}}$ (i.e. base of wall is in full soil contact)

$FS_{bc} = q_u / q_{max} = 5000 / 1423.4 = \mathbf{3.5} > 3.0$ **OK**

Graphical Solution

For low retaining walls a solution using the equivalent fluid pressure might be satisfactory. The equivalent fluid pressure is derived from Figure 4 and requires knowledge of the soil backfill. This Figure is presented for illustration only and the reader should refer to the referenced publication for details.

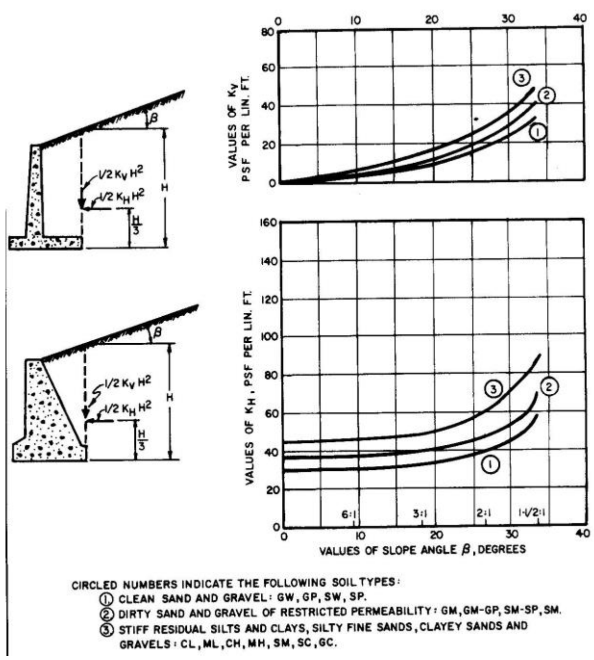


Figure 4 – Design Loads for Low Retaining Walls
[Ref: NAVFAC DM 7.2]

Important Points

The purpose of this course is to present basic subject matter to a diverse audience in order to convey the general concepts used when calculating the active earth pressure force and various factors of safety. The reader should understand the following:

- The Rankine and Coulomb methods are commonly used to calculate the active earth pressure force. The discussion in this course is limited to granular (cohesionless) backfill soil, which is a typical condition relating to retaining walls.

- The active earth pressure force (Pa) is a function of the earth pressure coefficient (Ka), the unit weight of the soil and the height of the wall.

- Wall movement must occur in order to develop the full active earth pressure force.

- Other lateral forces are superimposed on the lateral earth pressure force to derive the total lateral force.

- Retaining wall design is iterative and seeks to provide wall geometry that produces suitable factors of safety for sliding, overturning and bearing capacity.

- Retaining walls must also be checked for tolerable settlement and global stability.

Disclaimer

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References

1. Weber, Richard P., Personal Course Notes
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3. Department of the Navy, NAVFAC, DM-7.2, May 1982.