Department of Energy
Fundamentals Handbook

NUCLEAR PHYSICS AND REACTOR THEORY
Module 2
Reactor Theory (Neutron Characteristics)

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NEUTRON SOURCES

Neutrons from a variety of sources are always present in a reactor core. This is true even when the reactor is shut down. Some of these neutrons are produced by naturally occurring (intrinsic) neutron sources, while others may be the result of fabricated (installed) neutron sources that are incorporated into the design of the reactor. The neutrons produced by sources other than neutron-induced fission are often grouped together and classified as source neutrons.

EO 1.1 DEFINE the following terms:

a. Intrinsic neutron source
b. Installed neutron source

EO 1.2 LIST three examples of reactions that produce neutrons in intrinsic neutron sources.

EO 1.3 LIST three examples of reactions that produce neutrons in installed neutron sources.

Neutron Sources

In addition to neutron-induced fission, neutrons are produced by other reactions. The neutrons produced by reactions other than neutron-induced fission are called *source* neutrons. Source neutrons are important because they ensure that the neutron population remains high enough to allow a visible indication of neutron level on the most sensitive monitoring instruments while the reactor is shutdown and during the startup sequence. This verifies instrument operability and allows monitoring of neutron population changes. Source neutrons can be classified as either intrinsic or installed neutron sources.

Intrinsic Neutron Sources

Some neutrons will be produced in the materials present in the reactor due to a variety of unavoidable reactions that occur because of the nature of these materials. *Intrinsic neutron sources* are those neutron-producing reactions that always occur in reactor materials.

A limited number of neutrons will always be present, even in a reactor core that has never been operated, due to spontaneous fission of some heavy nuclides that are present in the fuel. Uranium-238, uranium-235, and plutonium-239 undergo spontaneous fission to a limited extent. Uranium-238, for example, yields almost 60 neutrons per hour per gram. Table 1 illustrates a comparison of the rate at which different heavy nuclides produce neutrons by spontaneous fission. Californium-252 is not an intrinsic neutron source, but will be discussed in the section on installed neutron sources.

TABLE 1
Neutron Production by Spontaneous Fission

Nuclide	$T_{1/2}$ (Fission)	$T_{1/2}$ (α-decay)	neutrons/sec/gram
$^{235}_{92}U$	1.8×10^{17} years	6.8×10^{8} years	8.0×10^{-4}
$^{238}_{92}U$	8.0×10^{15} years	4.5×10^{9} years	1.6×10^{-2}
$^{239}_{94}Pu$	5.5×10^{5} years	2.4×10^{4} years	3.0×10^{-2}
$^{240}_{94}Pu$	1.2×10^{11} years	6.6×10^{3} years	1.0×10^{3}
$^{252}_{98}Cf$	66.0 years	2.65 years	2.3×10^{12}

Another intrinsic neutron source is a reaction involving natural boron and fuel. In some reactors, natural boron is loaded into the reactor core as a neutron absorber to improve reactor control or increase core life-time. Boron-11 (80.1% of natural boron) undergoes a reaction with the alpha particle emitted by the radioactive decay of heavy nuclides in the fuel to yield a neutron as shown below.

$$^{11}_{5}B + ^{4}_{2}\alpha \rightarrow ^{14}_{7}N + ^{1}_{0}n$$

The boron-11 must be mixed with, or in very close proximity to, the fuel for this reaction because of the short path length of the alpha particle. For a reactor core with this configuration, this (α,n) reaction is an important source of neutrons for reactor startup.

In a reactor that has been operated, another source of neutrons becomes significant. Neutrons may be produced by the interaction of a gamma ray and a deuterium nucleus. This reaction is commonly referred to as a photoneutron reaction because it is initiated by electromagnetic radiation and results in the production of a neutron. The photoneutron reaction is shown below.

$$^{2}_{1}H + ^{0}_{0}\gamma \rightarrow \left(^{2}_{1}H\right)^{*} \rightarrow ^{1}_{1}H + ^{1}_{0}n$$

There is an abundant supply of high energy gammas in a reactor that has been operated because many of the fission products are gamma emitters. All water-cooled reactors have some deuterium present in the coolant in the reactor core because a small fraction of natural hydrogen is the isotope deuterium. The atom percentage of deuterium in the water ranges from close to the naturally occurring value (0.015%) for light water reactors to above 90% deuterium for heavy water reactors. Therefore, the required conditions for production of photoneutrons exist. The supply of gamma rays decreases with time after shutdown as the gamma emitters decay; therefore, the photoneutron production rate also decreases. In a few particular reactors, additional D_2O (heavy water) may be added to the reactor to increase the production of photoneutrons following a long shutdown period.

Installed Neutron Sources

Because intrinsic neutron sources can be relatively weak or dependent upon the recent power history of the reactor, many reactors have artificial sources of neutrons installed. These neutron sources ensure that shutdown neutron levels are high enough to be detected by the nuclear instruments at all times. This provides a true picture of reactor conditions and any change in these conditions. An *installed neutron source* is an assembly placed in or near the reactor for the sole purpose of producing source neutrons.

One strong source of neutrons is the artificial nuclide californium-252, which emits neutrons at the rate of about 2 x 10^{12} neutrons per second per gram as the result of spontaneous fission. Important drawbacks for some applications may be its high cost and its short half-life (2.65 years).

Many installed neutron sources use the (α,n) reaction with beryllium. These sources are composed of a mixture of metallic beryllium (100% beryllium-9) with a small quantity of an alpha particle emitter, such as a compound of radium, polonium, or plutonium. The reaction that occurs is shown below.

$$^{9}_{4}\mathrm{Be} + ^{4}_{2}\alpha \rightarrow \left(^{13}_{6}\mathrm{C}\right)^{*} \rightarrow ^{12}_{6}\mathrm{C} + ^{1}_{0}\mathrm{n}$$

The beryllium is intimately (homogeneously) mixed with the alpha emitter and is usually enclosed in a stainless steel capsule.

Another type of installed neutron source that is widely used is a photoneutron source that employs the (γ,n) reaction with beryllium. Beryllium is used for photoneutron sources because its stable isotope beryllium-9 has a weakly attached last neutron with a binding energy of only 1.66 MeV. Thus, a gamma ray with greater energy than 1.66 MeV can cause neutrons to be ejected by the (γ,n) reaction as shown below.

$$^{9}_{4}\mathrm{Be} + ^{0}_{0}\gamma \rightarrow \left(^{9}_{4}\mathrm{Be}\right)^{*} \rightarrow ^{8}_{4}\mathrm{Be} + ^{1}_{0}\mathrm{n}$$

Many startup sources of this type use antimony and beryllium because after activation with neutrons the radioactive antimony becomes an emitter of high energy gammas. The photoneutron sources of this type are constructed somewhat differently from the (α,n) types. One design incorporates a capsule of irradiated antimony enclosed in a beryllium sleeve. The entire assembly is then encased in a stainless steel cladding. A large reactor may have several neutron sources of this type installed within the core.

Summary

The important information in this chapter is summarized below.

Neutron Sources Summary

- Intrinsic neutron sources are sources of neutrons from materials that are in the reactor for other purposes such as fuel, burnable poison, or moderator.

- Installed neutron sources are materials or components placed in the reactor specifically for the purpose of producing source neutrons.

- Examples of intrinsic neutron sources are listed below.

  Spontaneous fission of heavy nuclides in fuel, such as uranium-238, uranium-235, and plutonium-239, results in fission fragments and free neutrons.

  Boron-11 mixed with the fuel undergoes an alpha-neutron reaction and becomes nitrogen-14.

  Deuterium present in the reactor coolant undergoes a gamma-neutron reaction and becomes hydrogen-1.

- Examples of installed neutron sources are listed below.

  Spontaneous fission of californium-252 results in fission fragments and free neutrons.

  Beryllium-9 undergoes an alpha-neutron reaction (alpha from the decay of plutonium, polonium, or radium) and becomes carbon-12.

  Beryllium-9 undergoes a gamma-neutron reaction (high energy gamma from decay of antimony-124) and becomes beryllium-8.

NUCLEAR CROSS SECTIONS AND NEUTRON FLUX

To determine the frequency of neutron interactions, it is necessary to describe the availability of neutrons to cause interaction and the probability of a neutron interacting with material. The availability of neutrons and the probability of interaction are quantified by the neutron flux and nuclear cross section.

EO 2.1 DEFINE the following terms:

a. Atom density
b. Neutron flux
c. Microscopic cross section
d. Barn
e. Macroscopic cross section
f. Mean free path

EO 2.2 EXPRESS macroscopic cross section in terms of microscopic cross section.

EO 2.3 DESCRIBE how the absorption cross section of typical nuclides varies with neutron energy at energies below the resonance absorption region.

EO 2.4 DESCRIBE the cause of resonance absorption in terms of nuclear energy levels.

EO 2.5 DESCRIBE the energy dependence of resonance absorption peaks for typical light and heavy nuclei.

EO 2.6 EXPRESS mean free path in terms of macroscopic cross section.

EO 2.7 Given the number densities (or total density and component fractions) and microscopic cross sections of components, CALCULATE the macroscopic cross section for a mixture.

EO 2.8 CALCULATE a macroscopic cross section given a material density, atomic mass, and microscopic cross section.

EO 2.9 EXPLAIN neutron shadowing or self-shielding.

Introduction

Fission neutrons are born with an average energy of about 2 MeV. These fast neutrons interact with the reactor core materials in various absorption and scattering reactions. Collisions that result in scattering are useful in slowing neutrons to thermal energies. Thermal neutrons may be absorbed by fissile nuclei to produce more fissions or be absorbed in fertile material for conversion to fissionable fuel. Absorption of neutrons in structural components, coolant, and other non-fuel material results in the removal of neutrons without fulfilling any useful purpose.

To safely and efficiently operate a nuclear reactor it is necessary to predict the probability that a particular absorption or scattering reaction will occur. Once these probabilities are known, if the availability of neutrons can be determined, then the rate at which these nuclear reactions take place can be predicted.

Atom Density

One important property of a material is the atom density. The *atom density* is the number of atoms of a given type per unit volume of the material. To calculate the atom density of a substance use Equation (2-1).

$$\mathbf{N} = \frac{\rho \ \mathbf{N_A}}{\mathbf{M}} \tag{2-1}$$

where:

N = atom density (atoms/cm^3)
ρ = density (g/cm^3)
N_A = Avogadro's number (6.022×10^{23} atoms/mole)
M = gram atomic weight

Example:

A block of aluminum has a density of 2.699 g/cm^3. If the gram atomic weight of aluminum is 26.9815 g, calculate the atom density of the aluminum.

Solution:

$$N = \frac{\rho \; N_A}{M}$$

$$= \frac{2.699 \; \frac{g}{cm^3} \left(6.022 \; x \; 10^{23} \; \frac{atoms}{mole} \right)}{26.9815 \; \frac{g}{mole}}$$

$$= 6.024 \; x \; 10^{22} \; \frac{atoms}{cm^3}$$

Cross Sections

The probability of a neutron interacting with a nucleus for a particular reaction is dependent upon not only the kind of nucleus involved, but also the energy of the neutron. Accordingly, the absorption of a thermal neutron in most materials is much more probable than the absorption of a fast neutron. Also, the probability of interaction will vary depending upon the type of reaction involved.

The probability of a particular reaction occurring between a neutron and a nucleus is called the *microscopic cross section* (σ) of the nucleus for the particular reaction. This cross section will vary with the energy of the neutron. The microscopic cross section may also be regarded as the effective area the nucleus presents to the neutron for the particular reaction. The larger the effective area, the greater the probability for reaction.

Because the microscopic cross section is an area, it is expressed in units of area, or square centimeters. A square centimeter is tremendously large in comparison to the effective area of a nucleus, and it has been suggested that a physicist once referred to the measure of a square centimeter as being "as big as a barn" when applied to nuclear processes. The name has persisted and microscopic cross sections are expressed in terms of *barns*. The relationship between barns and cm^2 is shown below.

1 barn = 10^{-24} cm^2

Whether a neutron will interact with a certain volume of material depends not only on the microscopic cross section of the individual nuclei but also on the number of nuclei within that volume. Therefore, it is necessary to define another kind of cross section known as the macroscopic cross section (Σ). The *macroscopic cross section* is the probability of a given reaction occurring per unit travel of the neutron. Σ is related to the microscopic cross section (σ) by the relationship shown below.

$$\Sigma = N \sigma \qquad (2\text{-}2)$$

where:

Σ = macroscopic cross section (cm^{-1})
N = atom density of material ($atoms/cm^3$)
σ = microscopic cross-section (cm^2)

The difference between the microscopic and macroscopic cross sections is extremely important and is restated for clarity. The microscopic cross section (σ) represents the effective target area that a single nucleus presents to a bombarding particle. The units are given in barns or cm^2. The macroscopic cross section (Σ) represents the effective target area that is presented by all of the nuclei contained in 1 cm^3 of the material. The units are given as 1/cm or cm^{-1}.

A neutron interacts with an atom of the material it enters in two basic ways. It will either interact through a scattering interaction or through an absorption reaction. The probability of a neutron being absorbed by a particular atom is the microscopic cross section for absorption, σ_a. The probability of a neutron scattering off of a particular nucleus is the microscopic cross section for scattering, σ_s. The sum of the microscopic cross section for absorption and the microscopic cross section for scattering is the total microscopic cross section, σ_T.

$$\sigma_T = \sigma_a + \sigma_s$$

Both the absorption and the scattering microscopic cross sections can be further divided. For instance, the scattering cross section is the sum of the elastic scattering cross section (σ_{se}) and the inelastic scattering cross section (σ_{si}).

$$\sigma_s = \sigma_{se} + \sigma_{si}$$

The microscopic absorption cross section (σ_a) includes all reactions except scattering. However, for most purposes it is sufficient to merely separate it into two categories, fission (σ_f) and capture (σ_c). Radiative capture of neutrons was described in the Neutron Interactions chapter of Module 1.

$$\sigma_a = \sigma_f + \sigma_c$$

The variation of absorption cross sections with neutron energy is often complicated. For many elements the absorption cross sections are small, ranging from a fraction of a barn to a few barns for slow (or thermal) neutrons.

For a considerable number of nuclides of moderately high (or high) mass numbers, an examination of the variation of the absorption cross section with the energy of the incident neutron reveals the existence of three regions on a curve of absorption cross section versus neutron energy. This cross section is illustrated in Figure 1. First, the cross section decreases steadily with increasing neutron energy in a low energy region, which includes the thermal range (E < 1 eV). In this region the absorption cross section, which is often high, is inversely proportional to the velocity (v). This region is frequently referred to as the "1/v region," because the absorption cross section is proportional to 1/v, which is the reciprocal of neutron velocity. Following the 1/v region, there occurs the "resonance region" in which the cross sections rise sharply to high values called "resonance peaks" for neutrons of certain energies, and then fall again. These energies are called resonance energies and are a result of the affinity of the nucleus for neutrons whose energies closely match its discrete, quantum energy levels. That is, when the binding energy of a neutron plus the kinetic energy of the neutron are exactly equal to the amount required to raise a compound nucleus from its ground state to a quantum level, resonance absorption occurs. The following example problem further illustrates this point.

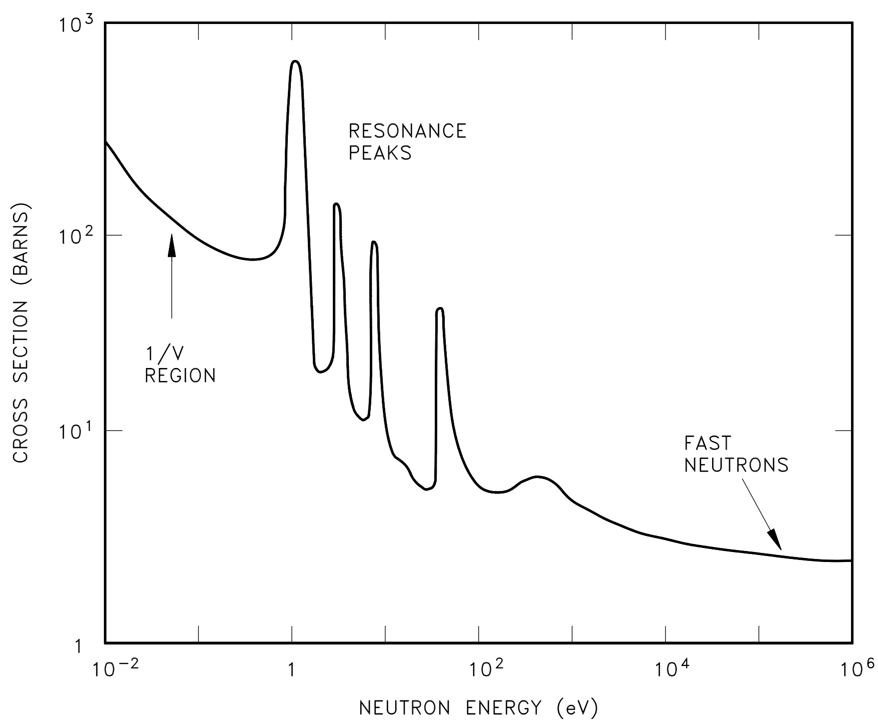

Figure 1 Typical Neutron Absorption Cross Section vs. Neutron Energy

Assuming that uranium-236 has a nuclear quantum energy level at 6.8 MeV above its ground state, calculate the kinetic energy a neutron must possess to undergo resonant absorption in uranium-235 at this resonance energy level.

$$BE = [\text{Mass}(^{235}U) + \text{Mass(neutron)} - \text{Mass}(^{236}U)] \times 931 \text{ MeV/amu}$$

$$BE = (235.043925 + 1.008665 - 236.045563) \times 931 \text{ MeV/amu}$$

$$BE = (0.007025 \text{ amu}) \times 931 \text{ MeV/amu} = 6.54 \text{ MeV}$$

$$6.8 \text{ MeV} - 6.54 \text{ MeV} = 0.26 \text{ MeV}$$

The difference between the binding energy and the quantum energy level equals the amount of kinetic energy the neutron must possess. The typical heavy nucleus will have many closely-spaced resonances starting in the low energy (eV) range. This is because heavy nuclei are complex and have more possible configurations and corresponding energy states. Light nuclei, being less complex, have fewer possible energy states and fewer resonances that are sparsely distributed at higher energy levels.

For higher neutron energies, the absorption cross section steadily decreases as the energy of the neutron increases. This is called the "fast neutron region." In this region the absorption cross sections are usually less than 10 barns.

With the exception of hydrogen, for which the value is fairly large, the elastic scattering cross sections are generally small, for example, 5 barns to 10 barns. This is close to the magnitude of the actual geometric cross sectional area expected for atomic nuclei. In potential scattering, the cross section is essentially constant and independent of neutron energy. Resonance elastic scattering and inelastic scattering exhibit resonance peaks similar to those associated with absorption cross sections. The resonances occur at lower energies for heavy nuclei than for light nuclei. In general, the variations in scattering cross sections are very small when compared to the variations that occur in absorption cross sections.

Mean Free Path

If a neutron has a certain probability of undergoing a particular interaction in one centimeter of travel, then the inverse of this value describes how far the neutron will travel (in the average case) before undergoing an interaction. This average distance traveled by a neutron before interaction is known as the *mean free path* for that interaction and is represented by the symbol λ. The relationship between the mean free path (λ) and the macroscopic cross section (Σ) is shown below.

$$\lambda = \frac{1}{\Sigma} \tag{2-3}$$

Calculation of Macroscopic Cross Section and Mean Free Path

Most materials are composed of several elements, and because most elements are composed of several isotopes, most materials involve many cross sections, one for each isotope involved. Therefore, to include all the isotopes within a given material, it is necessary to determine the macroscopic cross section for each isotope and then sum all the individual macroscopic cross sections. Equation (2-4) can be used to determine the macroscopic cross section for a composite material.

$$\Sigma = N_1 \sigma_1 + N_2 \sigma_2 + N_3 \sigma_3 + \ldots\ldots\ N_n \sigma_n \qquad (2\text{-}4)$$

where:

N_n = the number of nuclei per cm^3 of the n^{th} element
σ_n = the microscopic cross section of the n^{th} element

The following example problems illustrate the calculation of the macroscopic cross section for a single element and for combinations of materials.

Example 1:

Find the macroscopic thermal neutron absorption cross section for iron, which has a density of 7.86 g/cm^3. The microscopic cross section for absorption of iron is 2.56 barns and the gram atomic weight is 55.847 g.

Solution:

Step 1: Using Equation (2-1), calculate the atom density of iron.

$$N = \frac{\rho\ N_A}{M}$$

$$= \frac{7.86\ \frac{g}{cm^3}\left(6.022\ x\ 10^{23}\ \frac{atoms}{mole}\right)}{55.847\ \frac{g}{mole}}$$

$$= 8.48\ x\ 10^{22}\ \frac{atoms}{cm^3}$$

Step 2: Use this atom density in Equation (2-2) to calculate the macroscopic cross section.

$$\Sigma_a = N\ \sigma_a$$

$$= 8.48\ x\ 10^{22}\ \frac{atoms}{cm^3}\ (2.56\ barns)\left(\frac{1\ x\ 10^{-24}\ cm^2}{1\ barn}\right)$$

$$= 0.217\ cm^{-1}$$

Example 2:

An alloy is composed of 95% aluminum and 5% silicon (by weight). The density of the alloy is 2.66 g/cm^3. Properties of aluminum and silicon are shown below.

Element	Gram Atomic Weight	$\sigma_{a\ (barns)}$	$\sigma_{s\ (barns)}$
Aluminum	26.9815	0.23	1.49
Silicon	28.0855	0.16	2.20

1. Calculate the atom densities for the aluminum and silicon.

2. Determine the absorption and scattering macroscopic cross sections for thermal neutrons.

3. Calculate the mean free paths for absorption and scattering.

Solution:

Step 1: The density of the aluminum will be 95% of the total density. Using Equation (2-1) yields the atom densities.

$$N_{Al} = \frac{\rho_{Al}\ N_A}{M_{Al}}$$

$$= \frac{0.95 \left(2.66\ \frac{g}{cm^3}\right) \left(6.022\ x\ 10^{23}\ \frac{atoms}{mole}\right)}{26.9815\ \frac{g}{mole}}$$

$$= 5.64\ x\ 10^{22}\ \frac{atoms}{cm^3}$$

$$N_{Si} = \frac{\rho_{Si}\ N_A}{M_{Si}}$$

$$= \frac{0.05 \left(2.66\ \frac{g}{cm^3}\right) \left(6.022\ x\ 10^{23}\ \frac{atoms}{mole}\right)}{28.0855\ \frac{g}{mole}}$$

$$= 2.85\ x\ 10^{21}\ \frac{atoms}{cm^3}$$

Step 2: The macroscopic cross sections for absorption and scattering are calculated using Equation (2-4).

$$\Sigma_a = N_{Al}\ \sigma_{a,Al} + N_{Si}\ \sigma_{a,Si}$$
$$= \left(5.64 \text{ x } 10^{22}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(0.23 \text{ x } 10^{-24}\ \text{cm}^2\right) + \left(2.85 \text{ x } 10^{21}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(0.16 \text{ x } 10^{-24}\ \text{cm}^2\right)$$
$$= 0.0134\ \text{cm}^{-1}$$

$$\Sigma_s = N_{Al}\ \sigma_{s,Al} + N_{Si}\ \sigma_{s,Si}$$
$$= \left(5.64 \text{ x } 10^{22}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(1.49 \text{ x } 10^{-24}\ \text{cm}^2\right) + \left(2.85 \text{ x } 10^{21}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(2.20 \text{ x } 10^{-24}\ \text{cm}^2\right)$$
$$= 0.0903\ \text{cm}^{-1}$$

Step 3: The mean free paths are calculated by inserting the macroscopic cross sections calculated above into Equation (2-3).

$$\lambda_a = \frac{1}{\Sigma_a}$$
$$= \frac{1}{0.01345\ \text{cm}^{-1}}$$
$$= 74.3\ \text{cm}$$

$$\lambda_s = \frac{1}{\Sigma_s}$$
$$= \frac{1}{0.0903\ \text{cm}^{-1}}$$
$$= 11.1\ \text{cm}$$

Thus, a neutron must travel an average of 74.3 cm to interact by absorption in this alloy, but it must travel only 11.1 cm to interact by scattering.

Effects of Temperature on Cross Section

As discussed, the microscopic absorption cross section varies significantly as neutron energy varies. The microscopic cross sections provided on most charts and tables are measured for a standard neutron velocity of 2200 meters/second, which corresponds to an ambient temperature of 68°F. Therefore, if our material is at a higher temperature, the absorption cross section will be lower than the value for 68°F, and any cross sections which involve absorption (for example, σ_a, σ_c, σ_f) must be corrected for the existing temperature.

The following formula is used to correct microscopic cross sections for temperature. Although the example illustrates absorption cross section, the same formula may be used to correct capture and fission cross sections.

$$\sigma = \sigma_o \left(\frac{T_o}{T} \right)^{1/2}$$

where:

σ = microscopic cross section corrected for temperature
σ_o = microscopic cross section at reference temperature (68°F or 20°C)
T_o = reference temperature (68°F) in degrees Rankine (°R) or Kelvin (°K)
T = temperature for which corrected value is being calculated

NOTE: When using this formula, all temperatures must be converted to °R or °K.

°R = °F + 460
°K = °C + 273

Example:

What is the value of σ_f for uranium-235 for thermal neutrons at 500°F? Uranium-235 has a σ_f of 583 barns at 68°F.

Solution:

$$\sigma_f = \sigma_{f,o} \left(\frac{T_o}{T} \right)^{1/2}$$

$$= 583 \text{ barns} \left(\frac{68^\circ F + 460}{500^\circ F + 460} \right)^{1/2}$$

$$= 432 \text{ barns}$$

Neutron Flux

Macroscopic cross sections for neutron reactions with materials determine the probability of one neutron undergoing a specific reaction per centimeter of travel through that material. If one wants to determine how many reactions will actually occur, it is necessary to know how many neutrons are traveling through the material and how many centimeters they travel each second.

It is convenient to consider the number of neutrons existing in one cubic centimeter at any one instant and the total distance they travel each second while in that cubic centimeter. The number of neutrons existing in a cm^3 of material at any instant is called *neutron density* and is represented by the symbol n with units of neutrons/cm^3. The total distance these neutrons can travel each second will be determined by their velocity.

A good way of defining *neutron flux* (ϕ) is to consider it to be the total path length covered by all neutrons in one cubic centimeter during one second. Mathematically, this is the equation below.

$$\phi = n\,v \qquad (2\text{-}5)$$

where:

ϕ = neutron flux (neutrons/cm^2-sec)
n = neutron density (neutrons/cm^3)
v = neutron velocity (cm/sec)

The term neutron flux in some applications (for example, cross section measurement) is used as parallel beams of neutrons traveling in a single direction. The *intensity* (I) of a neutron beam is the product of the neutron density times the average neutron velocity. The directional beam intensity is equal to the number of neutrons per unit area and time (neutrons/cm^2-sec) falling on a surface perpendicular to the direction of the beam.

One can think of the neutron flux in a reactor as being comprised of many neutron beams traveling in various directions. Then, the neutron flux becomes the scalar sum of these directional flux intensities (added as numbers and not vectors), that is, $\phi = I_1 + I_2 + I_3 + ... I_n$. Since the atoms in a reactor do not interact preferentially with neutrons from any particular direction, all of these directional beams contribute to the total rate of reaction. In reality, at a given point within a reactor, neutrons will be traveling in all directions.

Self-Shielding

In some locations within the reactor, the flux level may be significantly lower than in other areas due to a phenomenon referred to as *neutron shadowing* or *self-shielding*. For example, the interior of a fuel pin or pellet will "see" a lower average flux level than the outer surfaces since an appreciable fraction of the neutrons will have been absorbed and therefore cannot reach the interior of the fuel pin. This is especially important at resonance energies, where the absorption cross sections are large.

Summary

The important information in this chapter is summarized below.

Nuclear Cross Section and Neutron Flux Summary

- Atom density (N) is the number of atoms of a given type per unit volume of material.
- Microscopic cross section (σ) is the probability of a given reaction occurring between a neutron and a nucleus.
- Microscopic cross sections are measured in units of barns, where 1 barn = 10^{-24} cm^2.
- Macroscopic cross section (Σ) is the probability of a given reaction occurring per unit length of travel of the neutron. The units for macroscopic cross section are cm^{-1}.
- The mean free path (λ) is the average distance that a neutron travels in a material between interactions.
- Neutron flux (ϕ) is the total path length traveled by all neutrons in one cubic centimeter of material during one second.
- The macroscopic cross section for a material can be calculated using the equation below.

$$\Sigma = N \sigma$$

- The absorption cross section for a material usually has three distinct regions. At low neutron energies (<1 eV) the cross section is inversely proportional to the neutron velocity.
- Resonance absorption occurs when the sum of the kinetic energy of the neutron and its binding energy is equal to an allowed nuclear energy level of the nucleus.
- Resonance peaks exist at intermediate energy levels. For higher neutron energies, the absorption cross section steadily decreases as the neutron energy increases.
- The mean free path equals 1/Σ.
- The macroscopic cross section for a mixture of materials can be calculated using the equation below.

$$\Sigma = N_1 \sigma_1 + N_2 \sigma_2 + N_3 \sigma_3 + \ldots\ldots. N_n \sigma_n$$

- Self-shielding is where the local neutron flux is depressed within a material due to neutron absorption near the surface of the material.

REACTION RATES

It is possible to determine the rate at which a nuclear reaction will take place based on the neutron flux, cross section for the interaction, and atom density of the target. This relationship illustrates how a change in one of these items affects the reaction rate.

EO 2.10 Given the neutron flux and macroscopic cross section, CALCULATE the reaction rate.

EO 2.11 DESCRIBE the relationship between neutron flux and reactor power.

Reaction Rates

If the total path length of all the neutrons in a cubic centimeter in a second is known, (neutron flux (ϕ)), and if the probability of having an interaction per centimeter path length is also known (macroscopic cross section (Σ)), multiply them together to get the number of interactions taking place in that cubic centimeter in one second. This value is known as the reaction rate and is denoted by the symbol R. The reaction rate can be calculated by the equation shown below.

$$R = \phi \Sigma \tag{2-6}$$

where:

R = reaction rate (reactions/sec)
ϕ = neutron flux (neutrons/cm^2-sec)
Σ = macroscopic cross section (cm^{-1})

Substituting the fact that $\Sigma = N \sigma$ into Equation (2-6) yields the equation below.

$$\mathbf{R} = \phi \, \mathbf{N} \, \sigma$$

where:

σ = microscopic cross section (cm^2)
N = atom density (atoms/cm^3)

The reaction rate calculated will depend on which macroscopic cross section is used in the calculation. Normally, the reaction rate of greatest interest is the fission reaction rate.

Example:

If a one cubic centimeter section of a reactor has a macroscopic fission cross section of 0.1 cm^{-1}, and if the thermal neutron flux is 10^{13} neutrons/cm^2-sec, what is the fission rate in that cubic centimeter?

Solution:

$$R_f = \phi \, \Sigma_f$$

$$= \left(1 \text{ x } 10^{13} \; \frac{\text{neutrons}}{\text{cm}^2\text{-sec}} \right) \left(0.1 \text{ cm}^{-1} \right)$$

$$= 1 \text{ x } 10^{12} \; \frac{\text{fissions}}{\text{cm}^3\text{-sec}}$$

In addition to using Equation (2-6) to determine the reaction rate based on the physical properties of material, it is also possible to algebraically manipulate the equation to determine physical properties if the reaction rate is known.

Example:

A reactor operating at a flux level of 3 x 10^{13} neutrons/cm^2-sec contains 10^{20} atoms of uranium-235 per cm^3. The reaction rate is 1.29 x 10^{12} fission/cm^3. Calculate Σ_f and σ_f.

Solution:

Step 1: The macroscopic cross section can be determined by solving Equation (2-6) for Σ_f and substituting the appropriate values.

$$R_f = \phi \, \Sigma_f$$

$$\Sigma_f = \frac{R_f}{\phi}$$

$$= \frac{1.29 \text{ x } 10^{12} \; \frac{\text{fissions}}{\text{cm}^3\text{- sec}}}{3 \text{ x } 10^{13} \; \frac{\text{neutrons}}{\text{cm}^2\text{- sec}}}$$

$$= 0.043 \text{ cm}^{-1}$$

Step 2: To find the microscopic cross section, replace Σ_f with (N x σ_f) and solve for σ_f.

$$R_f = \phi \; N \; \sigma_f$$

$$\sigma_f = \frac{R_f}{N \; \phi}$$

$$= \frac{1.29 \times 10^{12} \; \frac{\text{fissions}}{\text{cm}^3 - \text{sec}}}{\left(1 \times 10^{20} \; \frac{\text{atoms}}{\text{cm}^3}\right) \left(3 \times 10^{13} \; \frac{\text{neutrons}}{\text{cm}^2 - \text{sec}}\right)}$$

$$= 4.3 \times 10^{-22} \; \text{cm}^2 \left(\frac{1 \; \text{barn}}{1 \times 10^{-24} \; \text{cm}^2}\right)$$

$$= 430 \; \text{barns}$$

Reactor Power Calculation

Multiplying the reaction rate per unit volume by the total volume of the core results in the total number of reactions occurring in the core per unit time. If the amount of energy involved in each reaction were known, it would be possible to determine the rate of energy release (power) due to a certain reaction.

In a reactor where the average energy per fission is 200 MeV, it is possible to determine the number of fissions per second that are necessary to produce one watt of power using the following conversion factors.

1 fission = 200 MeV
1 MeV = 1.602 x 10^{-6} ergs
1 erg = 1 x 10^{-7} watt-sec

$$1 \; \text{watt} \left(\frac{1 \; \text{erg}}{1 \times 10^{-7} \; \text{watt} - \text{sec}}\right) \left(\frac{1 \; \text{MeV}}{1.602 \times 10^{-6} \text{erg}}\right) \left(\frac{1 \; \text{fission}}{200 \; \text{MeV}}\right) = 3.12 \times 10^{10} \; \frac{\text{fissions}}{\text{second}}$$

This is equivalent to stating that 3.12 x 10^{10} fissions release 1 watt-second of energy.

The power released in a reactor can be calculated based on Equation (2-6). Multiplying the reaction rate by the volume of the reactor results in the total fission rate for the entire reactor. Dividing by the number of fissions per watt-sec results in the power released by fission in the reactor in units of watts. This relationship is shown mathematically in Equation (2-7) below.

$$P = \frac{\phi_{th} \ \Sigma_f \ V}{3.12 \ x \ 10^{10} \ \frac{\text{fissions}}{\text{watt-sec}}} \tag{2-7}$$

where:

P = power (watts)
ϕ_{th} = thermal neutron flux (neutrons/cm^2-sec)
Σ_f = macroscopic cross section for fission (cm^{-1})
V = volume of core (cm^3)

Relationship Between Neutron Flux and Reactor Power

In an operating reactor the volume of the reactor is constant. Over a relatively short period of time (days or weeks), the number density of the fuel atoms is also relatively constant. Since the atom density and microscopic cross section are constant, the macroscopic cross section must also be constant. Examining Equation (2-7), it is apparent that if the reactor volume and macroscopic cross section are constant, then the reactor power and the neutron flux are directly proportional. This is true for day-to-day operation. The neutron flux for a given power level will increase very slowly over a period of months due to the burnup of the fuel and resulting decrease in atom density and macroscopic cross section.

Summary

The important information in this chapter is summarized below.

Reaction Rates Summary

- The reaction rate is the number of interactions of a particular type occurring in a cubic centimeter of material in a second.
- The reaction rate can be calculated by the equation below.

  $$R = \phi \, \Sigma$$

- Over a period of several days, while the atom density of the fuel can be considered constant, the neutron flux is directly proportional to reactor power.

NEUTRON MODERATION

In thermal reactors, the neutrons that cause fission are at a much lower energy than the energy level at which they were born from fission. In this type of reactor, specific materials must be included in the reactor design to reduce the energy level of the neutrons in an efficient manner.

EO 2.12 DEFINE the following concepts:

a. Thermalization
b. Moderator
c. Moderating ratio
d. Average logarithmic energy decrement
e. Macroscopic slowing down power

EO 2.13 LIST three desirable characteristics of a moderator.

EO 2.14 Given an average fractional energy loss per collision, CALCULATE the energy loss after a specified number of collisions.

Neutron Slowing Down and Thermalization

Fission neutrons are produced at an average energy level of 2 MeV and immediately begin to slow down as the result of numerous scattering reactions with a variety of target nuclei. After a number of collisions with nuclei, the speed of a neutron is reduced to such an extent that it has approximately the same average kinetic energy as the atoms (or molecules) of the medium in which the neutron is undergoing elastic scattering. This energy, which is only a small fraction of an electron volt at ordinary temperatures (0.025 eV at 20°C), is frequently referred to as the thermal energy, since it depends upon the temperature. Neutrons whose energies have been reduced to values in this region (< 1 eV) are designated thermal neutrons. The process of reducing the energy of a neutron to the thermal region by elastic scattering is referred to as *thermalization*, slowing down, or moderation. The material used for the purpose of thermalizing neutrons is called a *moderator*. A good moderator reduces the speed of neutrons in a small number of collisions, but does not absorb them to any great extent. Slowing the neutrons in as few collisions as possible is desirable in order to reduce the amount of neutron leakage from the core and also to reduce the number of resonance absorptions in non-fuel materials. Neutron leakage and resonance absorption will be discussed in the next module.

The ideal moderating material (moderator) should have the following nuclear properties.

- large scattering cross section
- small absorption cross section
- large energy loss per collision

A convenient measure of energy loss per collision is the logarithmic energy decrement. The *average logarithmic energy decrement* is the average decrease per collision in the logarithm of the neutron energy. This quantity is represented by the symbol ξ (Greek letter xi).

$$\xi = \ln E_i - \ln E_f$$

$$\xi = \ln \left(\frac{E_i}{E_f} \right) \tag{2-8}$$

where:

ξ = average logarithmic energy decrement
E_i = average initial neutron energy
E_f = average final neutron energy

The symbol ξ is commonly called the average logarithmic energy decrement because of the fact that a neutron loses, on the average, a fixed fraction of its energy per scattering collision. Since the fraction of energy retained by a neutron in a single elastic collision is a constant for a given material, ξ is also a constant. Because it is a constant for each type of material and does not depend upon the initial neutron energy, ξ is a convenient quantity for assessing the moderating ability of a material.

The values for the lighter nuclei are tabulated in a variety of sources. The following commonly used approximation may be used when a tabulated value is not available.

$$\xi = \frac{2}{A + \frac{2}{3}}$$

This approximation is relatively accurate for mass numbers (A) greater than 10, but for some low values of A it may be in error by over three percent.

Since ξ represents the average logarithmic energy loss per collision, the total number of collisions necessary for a neutron to lose a given amount of energy may be determined by dividing ξ into the difference of the natural logarithms of the energy range in question. The number of collisions (N) to travel from any energy, E_{high}, to any lower energy, E_{low}, can be calculated as shown below.

$$N = \frac{\ln E_{high} - \ln E_{low}}{\xi}$$

$$= \frac{\ln \left(\frac{E_{high}}{E_{low}} \right)}{\xi}$$

Example:

How many collisions are required to slow a neutron from an energy of 2 MeV to a thermal energy of 0.025 eV, using water as the moderator? Water has a value of 0.948 for ξ.

Solution:

$$N = \frac{\ln \left(\frac{E_{high}}{E_{low}} \right)}{\xi}$$

$$= \frac{\ln \left(\frac{2 \text{ x } 10^6 \text{ eV}}{0.025 \text{ eV}} \right)}{0.948}$$

$$= 19.2 \text{ collisions}$$

Sometimes it is convenient, based upon information known, to work with an average fractional energy loss per collision as opposed to an average logarithmic fraction. If the initial neutron energy level and the average fractional energy loss per collision are known, the final energy level for a given number of collisions may be computed using the following formula.

$$E_N = E_o (1 - x)^N \tag{2-9}$$

where:

E_o = initial neutron energy
E_N = neutron energy after N collisions
x = average fractional energy loss per collision
N = number of collisions

Example:

If the average fractional energy loss per collision in hydrogen is 0.63, what will be the energy of a 2 MeV neutron after (a) 5 collisions? (b) 10 collisions?

Solution:

a)

$$E_N = E_o (1 - x)^N$$
$$E_5 = (2 \text{ x } 10^6 \text{ eV}) (1 - 0.63)^5$$
$$= 13.9 \text{ keV}$$

b)

$$E_N = E_o (1 - x)^N$$
$$E_{10} = (2 \text{ x } 10^6 \text{ eV}) (1 - 0.63)^{10}$$
$$= 96.2 \text{ eV}$$

Macroscopic Slowing Down Power

Although the logarithmic energy decrement is a convenient measure of the ability of a material to slow neutrons, it does not measure all necessary properties of a moderator. A better measure of the capabilities of a material is the macroscopic slowing down power. The *macroscopic slowing down power* (MSDP) is the product of the logarithmic energy decrement and the macroscopic cross section for scattering in the material. Equation (2-10) illustrates how to calculate the macroscopic slowing down power.

$$\text{MSDP} = \xi \, \Sigma_s \tag{2-10}$$

Moderating Ratio

Macroscopic slowing down power indicates how rapidly a neutron will slow down in the material in question, but it still does not fully explain the effectiveness of the material as a moderator. An element such as boron has a high logarithmic energy decrement and a good slowing down power, but it is a poor moderator because of its high probability of absorbing neutrons.

The most complete measure of the effectiveness of a moderator is the moderating ratio. The *moderating ratio* is the ratio of the macroscopic slowing down power to the macroscopic cross section for absorption. The higher the moderating ratio, the more effectively the material performs as a moderator. Equation (2-11) shows how to calculate the moderating ratio of a material.

$$MR = \frac{\xi \ \Sigma_s}{\Sigma_a} \qquad (2\text{-}11)$$

Moderating properties of different materials are compared in Table 2.

TABLE 2
Moderating Properties of Materials

Material	ξ	Number of Collisions to Thermalize	Macroscopic Slowing Down Power	Moderating Ratio
H_2O	0.927	19	1.425	62
D_2O	0.510	35	0.177	4830
Helium	0.427	42	9×10^{-6}	51
Beryllium	0.207	86	0.154	126
Boron	0.171	105	0.092	0.00086
Carbon	0.158	114	0.083	216

Summary

The important information in this chapter is summarized below.

Neutron Moderation Summary

- Thermalization is the process of reducing the energy level of a neutron from the energy level at which it is produced to an energy level in the thermal range.
- The moderator is the reactor material that is present for the purpose of thermalizing neutrons.
- Moderating ratio is the ratio of the macroscopic slowing down power to the macroscopic cross section for absorption.
- The average logarithmic energy decrement (ξ) is the average change in the logarithm of neutron energy per collision.
- Macroscopic slowing down power is the product of the average logarithmic energy decrement and the macroscopic cross section for scattering.
- There are three desirable characteristics of a moderator.

  1. large scattering cross section
  2. small absorption cross section
  3. large energy loss per collision

- The energy loss after a specified number of collisions can be calculated using the equation below.

$$E_N = E_o (1 - x)^N$$

PROMPT AND DELAYED NEUTRONS

Not all neutrons are released at the same time following fission. Most neutrons are released virtually instantaneously and are called prompt neutrons. A very small fraction of neutrons are released after the decay of fission products and are called delayed neutrons. Although delayed neutrons are a very small fraction of the total number of neutrons, they play an extremely important role in the control of the reactor.

EO 3.1 **STATE the origin of prompt neutrons and delayed neutrons.**

EO 3.2 **STATE the approximate fraction of neutrons that are born as delayed neutrons from the fission of the following nuclear fuels:**

a. Uranium-235
b. Plutonium-239

EO 3.3 **EXPLAIN the mechanism for production of delayed neutrons.**

EO 3.4 **EXPLAIN prompt and delayed neutron generation times.**

EO 3.5 **Given prompt and delayed neutron generation times and delayed neutron fraction, CALCULATE the average generation time.**

EO 3.6 **EXPLAIN the effect of delayed neutrons on reactor control.**

Neutron Classification

The great majority (over 99%) of the neutrons produced in fission are released within about 10^{-13} seconds of the actual fission event. These are called *prompt neutrons*. A small portion of fission neutrons are *delayed neutrons*, which are produced for some time after the fission process has taken place. The delayed neutrons are emitted immediately following the first beta decay of a fission fragment known as a delayed neutron precursor. An example of a delayed neutron precursor is bromine-87, shown below.

$$ {}^{87}_{35}Br \xrightarrow[55.9\ \text{sec}]{\beta^-} {}^{87}_{36}Kr \xrightarrow[\text{instantaneous}]{n} {}^{86}_{36}Kr_{\text{stable}} $$

For most applications, it is convenient to combine the known precursors into groups with appropriately averaged properties. These groups vary somewhat depending on the fissile material in use. Table 3 lists the characteristics for the six precursor groups resulting from thermal fission of uranium-235. The fraction of all neutrons that are produced by each of these precursors is called the delayed neutron fraction for that precursor. The total fraction of all neutrons born as delayed neutrons is called the *delayed neutron fraction* (β). The fraction of delayed neutrons produced varies depending on the predominant fissile nuclide in use. The delayed neutron fractions (β) for the fissile nuclides of most interest are as follows: uranium-233 (0.0026), uranium-235 (0.0065), uranium-238 (0.0148), and plutonium-239 (0.0021).

TABLE 3
Delayed Neutron Precursor Groups for Thermal Fission in Uranium-235

Group	Half-Life (sec)	Delayed Neutron Fraction	Average Energy (MeV)
1	55.7	0.00021	0.25
2	22.7	0.00142	0.46
3	6.2	0.00127	0.41
4	2.3	0.00257	0.45
5	0.61	0.00075	0.41
6	0.23	0.00027	-
Total	-	0.0065	-

Neutron Generation Time

The neutron generation time is the time required for neutrons from one generation to cause the fissions that produce the next generation of neutrons. The generation time for prompt neutrons (ℓ* - pronounced "ell-star") is the total time from birth to rebirth. Three time intervals are involved: (a) the time it takes a fast neutron to slow down to thermal energy, (b) the time the now thermal neutron exists prior to absorption in fuel, and (c) the time required for a fissionable nucleus to emit a fast neutron after neutron absorption.

Fast neutrons slow to thermal energies or leak out of the reactor in 10^{-4} seconds to 10^{-6} seconds, depending on the moderator. In water moderated reactors, thermal neutrons tend to exist for about 10^{-4} seconds before they are absorbed. Fission and fast neutron production following neutron absorption in a fissionable nucleus occurs in about 10^{-13} seconds. Thus, fast reactors have an ℓ* of about 10^{-6} seconds, while thermal reactors have an ℓ* of about 10^{-6} seconds + 10^{-4} seconds, which is about 10^{-4} seconds to 10^{-5} seconds.

On the other hand, the average generation time for the six delayed neutron groups is the total time from the birth of the fast neutron to the emission of the delayed neutron. Again, three time intervals are involved: (a) the time it takes a fast neutron to slow down to thermal energy, (b) the time the thermal neutron exists prior to absorption, and (c) the average time from neutron absorption to neutron emission by the six precursor groups. The average time for decay of precursors from uranium-235 is 12.5 seconds. The other terms in the delayed neutron generation time are insignificant when compared to this value, and the average delayed neutron generation time becomes ~ 12.5 seconds.

A neutron generation time in the range of 10^{-4} seconds to 10^{-5} seconds or faster could result in very rapid power excursions, and control would not be possible without the dependence upon delayed neutrons to slow down the rate of the reaction. The average generation time, and hence the rate that power can rise, is determined largely by the delayed neutron generation time. The following equation shows this mathematically.

$$\text{Time}_{\text{average}} = \text{Time}_{\text{prompt}} \, (1 - \beta) + \text{Time}_{\text{delayed}} \, (\beta) \tag{2-12}$$

Example:

Assume a prompt neutron generation time for a particular reactor of 5×10^{-5} seconds and a delayed neutron generation time of 12.5 seconds. If β is 0.0065, calculate the average generation time.

Solution:

$$\begin{aligned}\text{Time}_{\text{average}} &= \text{Time}_{\text{prompt}} \, (1 - \beta) + \text{Time}_{\text{delayed}} \, (\beta) \\ &= \left(5 \text{ x } 10^{-5} \text{ seconds}\right) \left(0.9935\right) + \left(12.5 \text{ seconds}\right) \left(0.0065\right) \\ &= 0.0813 \text{ seconds}\end{aligned}$$

This example demonstrates the effect delayed neutrons have on the neutron generation time and thus reactor control. If a reactor were to be operated in a sustained chain reaction using only prompt neutrons ($\beta = 0$), the generation time from the previous example would be about 5×10^{-5} seconds. However, by operating the reactor such that a 0.0065 fraction of neutrons are delayed, the generation life time is extended to 0.0813 seconds, providing time for adequate operator control. Therefore, although only a small fraction of the total neutron population, delayed neutrons are extremely important to the control and maintenance of a sustained fission chain reaction.

Summary

The important information in this chapter is summarized on the following page.

Prompt and Delayed Neutrons Summary

- Prompt neutrons are released directly from fission within 10^{-13} seconds of the fission event.

- Delayed neutrons are released from the decay of fission products that are called delayed neutron precursors. Delayed neutron precursors are grouped according to half-life. Half-lives vary from fractions of a second to almost a minute.

- The fraction of neutrons born as delayed neutrons is different for different fuel materials. Following are values for some common fuel materials.

Uranium-235	0.0065
Plutonium-239	0.0021

- Delayed neutrons are produced by a classification of fission products known as delayed neutron precursors. When a delayed neutron precursor undergoes a β^- decay, it results in an excited daughter nucleus which immediately ejects a neutron. Therefore, these delayed neutrons appear with a half-life of the delayed neutron precursor.

- The delayed neutron generation time is the total time from the birth of the fast neutron to the emission of the delayed neutron in the next generation. Delayed neutron generation times are dominated by the half-life of the delayed neutron precursor. The average delayed neutron generation time is about 12.5 seconds.

- A prompt neutron generation time is the sum of the amount of time it takes a fast neutron to thermalize, the amount of time the neutron exists as a thermal neutron before it is absorbed, and the amount of time between a fissionable nuclide absorbing a neutron and fission neutrons being released. Prompt neutron generation time is about 5×10^{-5} seconds.

- The average neutron generation time can be calculated from the prompt and delayed neutron generation times and the delayed neutron fraction using Equation (2-12).

$$\text{Time}_{\text{average}} = \text{Time}_{\text{prompt}} \, (1 - \beta) + \text{Time}_{\text{delayed}} \, (\beta)$$

- Delayed neutrons are responsible for the ability to control the rate at which power can rise in a reactor. If only prompt neutrons existed, reactor control would not be possible due to the rapid power changes.

NEUTRON FLUX SPECTRUM

The number of neutrons that exist at a given energy level varies. A plot of either the fraction of neutrons or the neutron flux at a given energy versus the energy level is called a neutron energy spectrum. The neutron energy spectrum varies widely for different types of reactors.

EO 4.1 STATE the average energy at which prompt neutrons are produced.

EO 4.2 DESCRIBE the neutron energy spectrum in the following reactors:

a. Fast reactor
b. Thermal reactor

EO 4.3 EXPLAIN the reason for the particular shape of the fast, intermediate, and slow energy regions of the neutron flux spectrum for a thermal reactor.

Prompt Neutron Energies

The neutrons produced by fission are high energy neutrons, and almost all fission neutrons have energies between 0.1 MeV and 10 MeV. The neutron energy distribution, or spectrum, may best be described by plotting the fraction of neutrons per MeV as a function of neutron energy, as shown in Figure 2. From this figure it can be seen that the most probable neutron energy is about 0.7 MeV. Also, from this data it can be shown that the average energy of fission neutrons is about 2 MeV. Figure 2 is the neutron energy spectrum for thermal fission in uranium-235. The values will vary slightly for other nuclides.

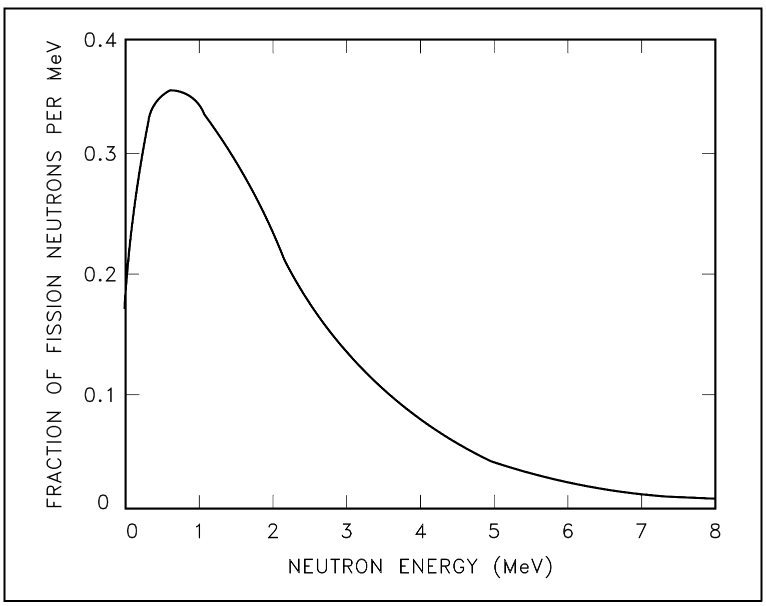

Figure 2 Prompt Fission Neutron Energy Spectrum for Thermal Fission of Uranium-235

Thermal and Fast Breeder Reactor Neutron Spectra

The spectrum of neutron energies produced by fission varies significantly from the energy spectrum, or flux, existing in a reactor at a given time. Figure 3 illustrates the difference in neutron flux spectra between a thermal reactor and a fast breeder reactor. The energy distribution of neutrons from fission is essentially the same for both reactors, so the differences in the curve shapes may be attributed to the neutron moderation or slowing down effects.

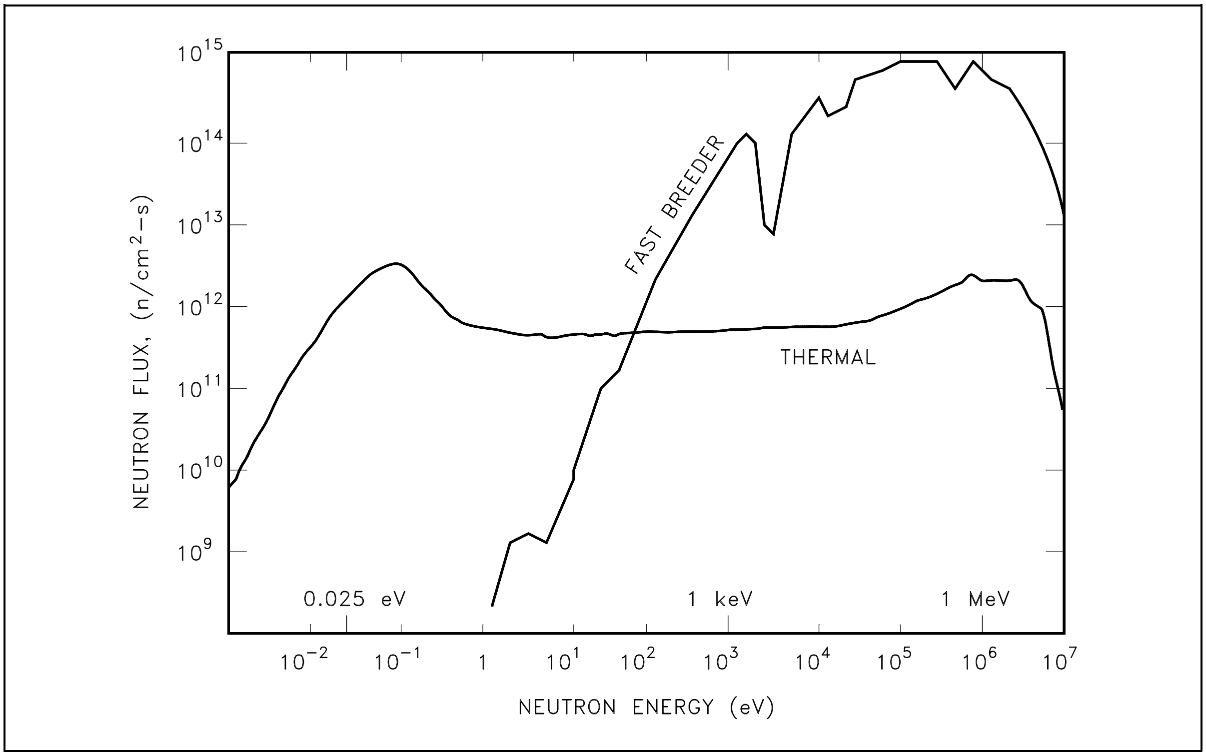

Figure 3 Comparison of Neutron Flux Spectra for Thermal and Fast Breeder Reactor

No attempt is made to thermalize or slow down neutrons in the fast breeder reactor (liquid metal cooled); therefore, an insignificant number of neutrons exist in the thermal range. For the thermal reactor (water moderated), the spectrum of neutrons in the fast region (> 0.1 MeV) has a shape similar to that for the spectrum of neutrons emitted by the fission process.

In the thermal reactor, the flux in the intermediate energy region (1 eV to 0.1 MeV) has approximately a 1/E dependence. That is, if the energy (E) is halved, the flux doubles. This 1/E dependence is caused by the slowing down process, where elastic collisions remove a constant fraction of the neutron energy per collision (on the average), independent of energy; thus, the neutron loses larger amounts of energy per collision at higher energies than at lower energies. The fact that the neutrons lose a constant fraction of energy per collision causes the neutrons to tend to "pile up" at lower energies, that is, a greater number of neutrons exist at the lower energies as a result of this behavior.

In the thermal region the neutrons achieve a thermal equilibrium with the atoms of the moderator material. In any given collision they may gain or lose energy, and over successive collisions will gain as much energy as they lose. These thermal neutrons, even at a specific temperature, do not all have the same energy or velocity; there is a distribution of energies, usually referred to as the Maxwell distribution (e.g., Figure 2). The energies of most thermal neutrons lie close to the most probable energy, but there is a spread of neutrons above and below this value.

Most Probable Neutron Velocities

The *most probable velocity* (v_p) of a thermal neutron is determined by the temperature of the medium and can be determined by Equation (2-13) .

$$v_p = \sqrt{\frac{2\ k\ T}{m}} \tag{2-13}$$

where:

v_p = most probable velocity of neutron (cm/sec)
k = Boltzman's constant (1.38×10^{-16} erg/°K)
T = absolute temperature in degrees Kelvin (°K)
m = mass of neutron (1.66×10^{-24} grams)

Example:

Calculate the most probable velocities for neutrons in thermal equilibrium with their surroundings at the following temperatures. a) 20°C, b) 260°C.

Solution:

a) Calculate the most probable velocity for 20°C using Equation (2-13).

$$v_p = \sqrt{\frac{2\ k\ T}{m}}$$

$$= \sqrt{\frac{2\left(1.38\ x\ 10^{-16}\ \frac{\text{erg}}{°\text{K}}\right)(293°\text{K})}{1.66\ x\ 10^{-24}\ \text{g}}}$$

$$= 2.2\ x\ 10^{5}\ \frac{\text{cm}}{\text{sec}}\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)$$

$$= 2200\ \frac{\text{m}}{\text{sec}}$$

b) Calculate the most probable velocity for 260°C using Equation (2-13).

$$v_p = \sqrt{\frac{2\ k\ T}{m}}$$

$$= \sqrt{\frac{2 \left(1.38\ x\ 10^{-16}\ \frac{erg}{°K}\right) (533°K)}{1.66\ x\ 10^{-24}\ g}}$$

$$= 2.977\ x\ 10^{5}\ \frac{cm}{sec} \left(\frac{1\ m}{100\ cm}\right)$$

$$= 2977\ \frac{m}{sec}$$

From these calculations it is evident that the most probable velocity of a thermal neutron increases as temperature increases. The most probable velocity at 20°C is of particular importance since reference data, such as nuclear cross sections, are tabulated for a neutron velocity of 2200 meters per second.

Summary

The important information in this chapter is summarized below.

Neutron Flux Spectrum Summary

- Prompt neutrons are born at energies between 0.1 MeV and 10 MeV. The average prompt neutron energy is about 2 MeV.
- Fast reactors have a neutron energy spectrum that has the same shape as the prompt neutron energy spectrum.
- Thermal reactors have a neutron energy spectrum that has two pronounced peaks, one in the thermal energy region where the neutrons are in thermal equilibrium with the core materials and another in the fast region at energies where neutrons are produced. The flux in the intermediate region (1 eV to 0.1 MeV) has a roughly 1/E dependence.
- The neutron flux spectrum for the fast energy region of a thermal reactor has a shape similar to that of the spectrum of neutrons emitted by the fission process.
- The reason for the 1/E flux dependence at intermediate energy levels in a thermal reactor is due to the neutrons' tendency to lose a constant fraction of energy per collision. Since the neutrons lose a greater amount at the higher energies, the neutrons tend to "pile up" at lower energies where they lose less energy per collision.
- The neutron flux spectrum for the slow region of a thermal reactor contains a peak at the energy where the neutrons are in thermal equilibrium with the atoms of the surrounding material.