Fundamentals of Reactive Power and Voltage Regulation in Power Systems

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**Introduction**

One of the main goals that every electrical utility company has is transportation of electrical energy from generating station to the customer, meeting the following main criteria:

- High reliability of power supply
- Low energy cost
- High quality of energy (required voltage level, frequency etc.)

This course is concentrated on accomplishing the 2\textsuperscript{nd} and 3\textsuperscript{rd} goals through regulation of reactive power and voltage. Reliability of power supply is a subject of a different course.

To better understand why the regulation of reactive power and voltage makes power systems more efficient, let’s start with discussion about the structure of the power systems and their main components.

**Power System Structure**

The typical power system structure is shown in Figure 1.

![Fig. 1 - Power System Structure and Main Components](image)

Where the numerical symbols represent the following components:

1. Generator
2. Generating station’s step-up transformer substation
3. Extra high voltage step-down transformer substation
4. High voltage step-down transformer substation
5. Distribution substation
6. Distribution Transformer
7. Transmission and Distribution Lines
8. Customer
The elements from 2 to 7 are the components of utility company Transmission and Distribution (T&D) Systems, with typical power system voltages as follows:

- Generation: Up to 25 kV
- Transmission: 115 – 1500 kV
- Subtransmission: 26 – 69 kV
- Distribution: 4 – 13 kV
- Customer: Up to 600 V

**Justification for Voltage Transformation**

As we can see from the Figure 1, along the route from the source to the customer, electricity is undergoing numerous transformations, with generating voltage getting stepped-up to transmission level with a follow-up decrease down to distribution and eventually customer levels. Why do we need to do it? As previously was mentioned, utility company wants to keep energy costs down. One of the ways to do it is to reduce power and energy losses, which may be accomplished by raising the voltage level, because power losses $\Delta P$ have a functional relationship with voltage level described in the following equation:

$$\Delta P = F \left( S^2 x L / V^2 \right)$$  \hspace{1cm} (1)

Where:

- $S$ is transported apparent power
- $L$ is distance to the customer
- $V$ is system voltage level

As we can see, there is a reverse proportion between power losses and voltage level in the 2\textsuperscript{nd} degree. For example, if we increase voltage 10 times, power losses will be 100 times smaller.

Another benefit from raising the voltage is a reduction of voltage drop, which is related to voltage level $\Delta V$ as follows:

$$\Delta V = F \left( S x L / V \right)$$  \hspace{1cm} (2)

Having smaller voltage drop in the system helps a utility company meet its other objective which is to provide customer with a high quality electrical energy meeting specific voltage level requirements.

That’s why we increase voltage for transmission of electrical energy, but after it is delivered to the area where customers are located, we gradually lower the voltage to the
safe utilization level (208/120 V, for example). Number of steps in raising and lowering the voltage is being defined through optimization studies performed by utility company Planners.

**Fundamentals of Reactive Power Regulation**

Besides changing the voltage level, there is another way to reduce power and energy losses through a reactive power regulation. Let’s see how it may be done.

An apparent power $S$ carried by a power line has two components active power $P$ and a reactive power $Q$, which are related as follows:

$$S = P + jQ = (P^2 + Q^2)^{1/2} e^{j \arctan Q/P}, \quad (3)$$

Where $\arctan Q/P = \varphi$, which is an angle between $P$ and $S$. Relationship between $P$, $Q$ and $S$ is shown in Figure 2 in so called “Power Triangle”. The ratio of $P$ to $S$ is called the power factor, which from Figure 2 is equal to $\cos \varphi$. For the inductive load, current looking counterclockwise lags the voltage and the power factor is correspondingly called “lagging”. For the capacitive load, current leads the voltage, and the power factor is called “leading”. For a 100% active load ($Q = 0$), $P = S \rightarrow \cos \varphi = 1$, and for a 100% reactive load ($P=0$), $\cos \varphi = 0$. Usually, a power system has a wide mix of customers with different shares of active and reactive loads, which makes the combined power factor $0 \leq \cos \varphi \leq 1$.

Fig. 2 - Relationship between $S$, $P$ and $Q
Active power losses $\Delta P$ and voltage drop $\Delta V$ may be found from the following equations:

$$\Delta P = \frac{(P^2 + Q^2) \times r}{V^2} \quad (4)$$

$$\Delta V = 3^{1/2} \times \frac{(P^2 + Q^2)^{1/2} \times r}{V} \quad (5)$$

Where:

$V$ is system voltage
$R$ is circuit’s resistance

As we can see from Equations (4) and (5) reduction of reactive power transported from generating station to the customers will lead to reduction of both active power losses and voltage drops. To achieve this goal, local sources of reactive power may be used: either shunt capacitors for inductive load, or shunt reactors for capacitive load. Let’s discuss both options.

**Shunt Capacitors**

As it was mentioned before, shunt capacitors may be used to provide a local source of capacitive reactive power $Q_C$ to reduce a value of inductive reactive power $Q_L$ carried by the line (usually, an overhead line). The results achieved by the application of shunt capacitors are shown in Figure 3.

![Fig. 3 - Application of Shunt Capacitors for Power Factor Improvement](image)

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The results achieved are as follows:

- Without a shunt capacitor, apparent power carried by the line \( S_L = P_L + jQ_L \), and power factor \( \cos \phi = \frac{P_L}{S_L} \).
- With a capacitor, line apparent power, \( S_{L1} = P_L + j(Q_L - Q_C) < S_L \), and power factor \( \cos \phi_1 = \frac{P_L}{S_{L1}} > \cos \phi \).
- Ultimately, power losses \( \Delta P \) and voltage drop \( \Delta V \) will be reduced after shunt capacitor is installed, i.e. \( \Delta P_1 < \Delta P \), and \( \Delta V_1 < \Delta V \).

Usually, shunt capacitors are coming as banks made up of a number of capacitor units that should be connected in parallel and series sections to obtain desired ratings of the bank, both voltage and capacity. One phase of the typical 26 kV capacitor bank is shown in Figure 4. It consists of six “cans” that are going to be connected as per manufacturer instructions.

Let’s learn how to design these connections. Figure 5 shows possible connections for one phase of a three-phase capacitor bank with capacitor units connected in parallel groups and groups connected in series. Another option is to connect units in series to form a...
string, and after this, connect strings in parallel. Combination of these two options is possible as well. For any connection scheme utilizing capacitor units rated for a voltage \( V_U \) and a reactive power \( Q_U \), the following equations may be used to calculate numbers of units in each phase required to obtain for the 3-phase bank a total power rating of \( Q_B \) at a system line voltage \( V_L \)

\[
N_{SER} = \frac{V_L}{3^{1/2} \times V_U} \quad N_{PAR} = \frac{Q_B}{3 \times N_{SER} \times Q_U}, \quad (6)
\]

Where:

- \( N_{SER} \) = number of series sections
- \( N_{PAR} \) = number of capacitor units connected in parallel in each series section

![Diagram of Capacitor Units Connection](image)

Fig. 5 - Connections of Capacitor Units into a Single Phase Bank

It should be noted, that the following relationship exists between a reactive power \( Q_U \) of every capacitor unit with a capacitance \( C_U \) and a voltage \( V_U \) connected to it:

\[
Q_U = C_U \times V_U^2 \quad (7)
\]

If \( N_{SER} \), found from the Equation (6), is not the whole number, the closest higher number of sections should be selected. A lower is not acceptable because the voltage per section in this case will be more than the rated voltage for which a capacitor unit is designed. As a result of such selection, the voltage \( V_{U}' \) per every series section will be lower than a rated voltage \( V_U \). From Equation (7), it follows that the reactive power \( Q_{U}' \) will be lower than \( Q_U \) as well. The following equation may be used to find the derating of a capacitor unit:

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\[ Q_U' = Q_U \times (V_U'/V_U)^2 \]  \hspace{1cm} (8)

Then, \( Q_U' \) should be used instead of \( Q_U \) in Equation (6) to find \( N_{\text{PAR}} \). The number of capacitor units that may be connected in parallel and series has some limitations described in the IEEE Guide for Protection of Shunt Capacitor Banks \([1]\). The minimum number of units in one series section is determined by overvoltage considerations, after one unit in the section fails. Table 2 of this IEEE Guide \([1]\) contains this minimum \( N_{\text{PAR}} \) for different values of \( N_{\text{SER}} \) and connection schemes of the whole bank.

At the same time, there is a maximum number of capacitors that may be connected in parallel. It is based on considerations of possible discharge transient from a large number of capacitors following the failure of one of them. This discharge may cause a rupture of the failed unit with possible damage to the rest of the bank. To prevent it, the maximum reactive power of one series section should not be higher than 4,650 kvar at a rated voltage and 60 Hz frequency. Refer to IEEE Std. C37.99-1990 “IEEE Guide for Protection of Shunt Capacitor Banks \([1]\).”

Example 1 shows, in a simplified form, a calculation of capacitive power required to improve a power factor up to a certain level and a selection of the number of capacitor units that should be connected together to provide this power.

**Example 1. Design of 230 kV shunt capacitor bank:**

For a 230 kV system with a power flow, shown in Figure 6, design a shunt capacitor bank that should be installed at substation to increase \( \cos \phi \) up to at least 0.9 (lagging). The 3-phase bank should be built using capacitor units rated 13.28 kV, 200 kvar. Phases should be connected in a grounded wye.

Fig. 6 - Power Flow for Example 1

\[ V_L = 230 \text{ kV} \]

\[ S_{\text{Lk}} = 200 + j60 \]

\[ S_{\text{out}} = 125 + j90 \]

\[ S_{\text{CB}} \]

\[ S_{\text{load}} = 75 + j60 \]
Where:

- $S_{\text{in}}$ is Incoming power, MVA
- $S_{\text{out}}$ is Outgoing power, MVA
- $S_{\text{load}}$ is Load, MVA
- SCB is Shunt capacitor bank

Solution:

1. Find the existing power factor $\cos \phi_1$:

   \[
   \tan \phi_1 = \frac{Q_{\text{in}}}{P_{\text{in}}} = \frac{150}{200} = 0.75 \rightarrow \phi_1 = 37^\circ \rightarrow \cos \phi_1 = 0.8
   \]

2. Draw power triangle:

   ![Fig. 7 - Power Triangle for Example 1](image)

3. Find new angle $\phi_2$ corresponding $\cos \phi_2 = 0.9$:

   \[
   \phi_2 = 25.8^\circ
   \]

4. Find a new incoming reactive power $Q'_{\text{in}}$:

   \[
   Q'_{\text{in}} = P_{\text{in}} \times \tan \phi_2 = 200 \times \tan 25.8^\circ = 200 \times 0.484 = 96.86 \text{ mvar}
   \]

   It should be noted, that a value of the active power $P_{\text{in}}$ didn’t change with a change of $\cos \phi$, because it depended on a load demand that stayed the same.

5. Show $Q'_{\text{in}}$, $S'_{\text{in}}$ and $\phi_2$ on a power triangle (see Figure 7). A corresponding apparent power $S'_{\text{in}}$ may be written as:
$$S'_{in} = P_{in} + j Q'_{in} = (200 + j96.86) \text{ mVA}$$

6. Find a size of shunt capacitor bank:

$$Q_{SCB} = Q_{in} - Q'_{in} = 150 - 96.86 = 53.14 \text{ mvar}$$

7. Assuming for a capacitor bank connection option shown in Figure 5 and using Equation (6) find:

7.1. Number of series sections in each phase:

$$N_{SER} = 230 / (3^{1/2} \times 13.28) = 10.0$$

7.2. Number of parallel capacitor units in each series section:

$$N_{PAR} = Q_{PAR} / (3 \times 10 \times 0.2) = 8.85$$

The next higher number of 9 should be selected to meet a requirement of improving $\cos \phi$ up to at least 0.9.

From Table 2 of this IEEE Guide [1], the minimum number of parallel capacitors in each out of 10 series sections, for a grounded wye connection of the whole bank, is 10. Thus, $N_{PAR} = 10$ should be used.

The total reactive power $Q_{PAR}$ of each series section, consisting of 10 parallel capacitors, may be found as follows:

$$Q_{PAR} = Q_U \times N_{PAR} = 200 \times 10 = 2000 \text{ kvar}$$

That is less than a maximum of 4650 kvar, recommended by IEEE Std. C37.99-1990 “IEEE Guide for Protection of Shunt Capacitor Banks [1]. Thus, a selected number of 10 parallel capacitors in each series section is acceptable.

The total capacitive power of the whole bank is:

$$Q_{C'} = 0.2 \times 10 \times 10 \times 3 = 60 \text{ mvar}$$

The new reactive power, flowing into the substation, is:

$$Q_{in''} = Q_{in} - Q_{C'} = 150 - 60 = 90 \text{ mvar}$$

And the new power factor $\cos \phi_3$ may be found as follows:

$$\tan \phi_3 = Q_{in''} / P_{in} = 90/200 = 0.45, \quad \phi_3 = 24.22^\circ, \quad \cos \phi_3 = 0.912,$$
Thus, a designed three-phase capacitor bank provides the required improvement of $\cos \phi$ of the incoming power. Each phase of this bank consists of 10 series sections with 10 capacitors, connected in parallel in each section, with connections made, as shown in Figure 5.

Location of shunt capacitors in the power system is based on special optimization studies. They may be installed at the transmission and distribution substations as well as on distribution line poles. After optimal location for shunt capacitors is identified, substation engineer should select the proper equipment to switch and protect capacitor banks. Very often, there are several capacitor banks installed at the station and they may be switched in and out separately to obtain a desired level of compensation of reactive power in the system. This switching is usually done by properly rated circuit breakers or circuit switchers, depending on particular application and system requirements. Circuit breaker is rated to interrupt both load and fault current while circuit switcher is mostly used to switch load current only. These switching devices may be operated locally and remotely, giving system personnel an opportunity to disconnect capacitors from the substation bus when a situation in the system temporarily does not require additional reactive power compensation, and put them back in service when it is necessary. Examples of circuit breaker and circuit switcher are shown in Figures 8 and 9 respectively.

Fig. 8 - 230 kV 2000 A Circuit Breaker
So far, we’ve considered the application of shunt capacitors to compensate inductive reactive power, but in some power systems a reactive component Q of an apparent power S is mostly capacitive, i.e. a power factor is leading. Usually it occurs when transmission lines in power system are underground cables rather than overhead lines. Equation for the apparent power for capacitive systems will look as follows:

\[ S = P - jQ \quad (9) \]

Power triangle for a capacitive load is shown in Figure 10.
To reduce a value of a capacitive reactive power carried by transmission line in this system we need to use a shunt reactor as a local source of inductive reactive power. The results achieved are as follows:

- Without a shunt reactor, apparent power carried by the line \( S_L = P_L - jQ_L \) and power factor \( \cos \phi = \frac{P_L}{S_L} \)
- With a reactor, line apparent power, \( S_{L1} = P_L - j(Q_L - Q_R) < S_L \), and \( \cos \phi_1 = \frac{P_L}{S_{L1}} > \cos \phi \)
- Ultimately, power losses \( \Delta P \) and voltage drop \( \Delta V \) will be reduced after shunt reactor is installed, i.e. \( \Delta P_1 < \Delta P \), and \( \Delta V_1 < \Delta V \)

The reactive power rating of a shunt reactor may be found similarly to the selection of a shunt capacitor in example 1 (see Figure 11).

To improve a power factor from \( \cos \phi_1 \) to \( \cos \phi_2 \), the required reactor rating \( Q_R \) is:

\[
Q_R = Q_{in} - Q_{in}'
\]  

(10)

![Fig. 11 - Calculation of Shunt Reactor Rating](image)

An optimum location of shunt reactor in the system is defined by special studies. Usually it is installed at transmission substations. Construction of a shunt reactor is similar to the arrangement of a power transformer. The difference is that transformers have at least two windings, while a reactor has only one. The devices used to switch shunt reactor are circuit breakers and circuit switchers. The example of 230 kV shunt reactor is shown in Figure 12.
Previously we’ve discussed how to reduce power losses and voltage drops in power systems using compensation of reactive power with either shunt capacitors (for inductive load), or shunt reactors (for capacitive load). However, it is not the only way for utility company to meet their goal - to provide customer with electricity at the standard voltage level (208/120 V, for example). Another option to compensate for voltage drops in power lines and transformers is to use a voltage regulation. Let’s discuss the main methods used by power companies to do it.

One of the most common means of voltage regulation is having transformer windings with taps to change the turns ratio. A high voltage (26 – 230 kV) winding of a substation transformer usually has no-load taps that may be changed only manually when the transformer is deenergized. Because of this necessity to take transformer out of service, this method of voltage regulation is used infrequently, usually when there is a substantial load growth or seasonal load fluctuations.

Another way to regulate voltage is to have a load tap changer (LTC) on a low voltage (4 - 26 kV) winding of substation transformer, which is used for frequent changes in the transformer turns ratio while it is carrying the load. LTC may be operated manually or automatically, locally or remotely. For automatic operation, LTC has a control device that monitors voltage and changes taps to keep it within certain limits. The example of a transformer with LTC is shown in Figure 13.
The 3rd option for voltage regulation is an application of voltage regulators, which are installed on each substation feeder to change their voltage under load. The example of a voltage regulator is shown in Figure 14.

Fig. 13 - 26 kV LTC on 230/26 kV 55 MVA Transformer

Fig. 14 - 4 kV Voltage Regulators
It needs to be noted, that voltage regulation should not be done to raise voltage to a standard level for the very last customer on the line, but rather to keep voltage for all the customers inside required boundaries, for example, ±5%, which for 120V will amount to a 114V – 126V range. It is very important to keep voltage inside these boundaries because they are specified for all the appliances to have them functioning the way they are intended to. For example, if voltage in your house is too low, lights will dim making reading difficult. If voltage is too high, lights will be very bright, blinding the reader, and requiring frequent replacement of the lighting bulbs. For explanation of this principle of voltage regulation, let’s use Figure 15, showing distribution line supplying power from substation A to several customers B – F.

Voltage regulation is done at the substation either by changing transformer taps (under load or with no load) for all the substation feeders or by using voltage regulators just on the line A – F. In any case, voltage level at the substation should be installed high enough to compensate the voltage drop in the line under maximum load conditions. However, if regulation is based on keeping the voltage for customer F at standard level (120 V, for example), the voltage at substation A will be set so high that customer B will see voltage higher that allowed 5% over standard, for example 130 V. So, to make all the customers satisfied, the voltage level at substation should be set to have voltage at customer B no higher than 126 V and at customer F no lower than 114 V. If this can’t be accomplished by regulating voltage at the substation, additional capacitor banks may be placed between substation and customer F.

To understand how transformer taps may be selected, we’ll use Example 2.

**Example 2. Selection of no-load taps on the high side of 26/4 kV transformer**

A high voltage winding of a single phase 26.4/4.16 kV transformer has five no-load taps to keep secondary voltage constant by changing the turns ratio correspondingly to the changes in a primary voltage within ±5% in 2.5% increments (see Figure 16). Load flow studies show that for a maximum summer load, the expected voltage on 26.4 kV side will be 24.8 kV. Knowing that the lowest allowed limit for a secondary voltage is 4.08 kV select a corresponding primary no-load tap for the summer season.
Solution:

- We start with calculating primary transformer voltages for all five (5) taps. The results, which are shown in Table 1, will be used for all further iterations.

<table>
<thead>
<tr>
<th>HV Tap</th>
<th>Changes in Primary Voltage, %</th>
<th>Primary Voltage, V</th>
<th>Secondary Voltage, V</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 6</td>
<td>- 5</td>
<td>25,080</td>
<td>4,160</td>
</tr>
<tr>
<td>1 – 5</td>
<td>- 2.5</td>
<td>25,740</td>
<td>4,160</td>
</tr>
<tr>
<td>1 – 4</td>
<td>0</td>
<td>26,400</td>
<td>4,160</td>
</tr>
<tr>
<td>1 – 3</td>
<td>+2.5</td>
<td>27,060</td>
<td>4,160</td>
</tr>
<tr>
<td>1 – 2</td>
<td>+5.0</td>
<td>27,720</td>
<td>4,160</td>
</tr>
</tbody>
</table>

- For a middle high voltage tap 1 – 4 transformation factor:

  \[ N(0) = \frac{26,400}{4,160} = 6.346 \]

  During summer load maximum,

  \[ V_1 = 24.8 \text{ kV and } V_2 = \frac{V_1}{N(0)} = \frac{24.8}{6.346} = 3.91 < 4.08 \text{ kV.} \]

  So, requirement is not met and next lower tap should be selected.
• For the next lower tap 1 – 5:

\[
N(-2.5\%) = 25, \quad 740/4,160 = 6.188,
\]

\[
V''_2 = V_1/N(-2.5\%) = 24.8/6.188 = 4.0 < 4.08 \text{ kV}
\]

Again, requirement is not met and next lower tap should be selected.

• For the lowest tap 1 – 6,

\[
N(-5\%) = 25,080/4,160 = 6.03,
\]

\[
V'''_2 = V_1/N(-5.0\%) = 24.8/6.03 = 4.11 > 4.08 \text{ kV},
\]

This meets the requirement.

So, before summer season, the transformer should be deenergized and a tap 1 – 6 should be installed on a high voltage winding to ensure that secondary voltage meets requirements during a load maximum.

If the last iteration did not provide a needed result, additional means of voltage regulation could be used, like voltage regulators installed on distribution feeders, shunt capacitors etc.

**Conclusion**

This course provided an overview of fundamentals of reactive power and voltage regulation to enable you to:

• Understand why reactive power needs to be regulated
• Draw power triangle
• List main equipment for reactive power regulation
• Size and design shunt capacitor bank for a specific power factor improvement
• Describe the mission of voltage regulation and means used to perform it
• For a specific limits of secondary voltage select a no-load taps on primary side of transformer

**References**