Fuzzy Logic:  
*An Introduction to Fuzziness in Controllers*

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INTRODUCTION

Human beings have the ability to take in and evaluate all sorts of information from the physical world and mentally analyze and summarize all this input data into an optimum course of action. All living things do this, but humans do it more and do it better and have become the dominant species of the planet.

Computers, on the other hand, operate on a binary true or false basis. Unfortunately our world is not binary. The world we live in is full of ambiguities. The way a computer operates and how our world functions appear to be odd if we try to have computers mimic worldly events. "The temperature is pretty warm" cannot be evaluated as strictly true or false. We accept that this statement has certain ambiguities. As the complexity of a system increases, it becomes more difficult and, eventually, impossible to make a precise statement about its behavior. This eventually leads to a point of complexity where the fuzzy logic method born in humans is the only way to get at the problem.

Thus, the mathematical theory of fuzzy logic was developed. The theory of fuzzy logic states that rather than a statement being true or false, each statement has a certain confidence level. For example let us say a confidence value of 0.000 meant false and a confidence value 1.000 meant true, then the statement "this room is warm" might have a confidence value of .700 at 80 degrees. The idea of fuzzy logic has had a profound impact in many areas including decision making software in the control arena. Specifically, there are applications where “fuzziness” is utilized when human lives are at stake.

If you think about it, much of the information you take in is not very precisely defined, such as evaluation of the behavior of a vehicle entering from a side street and the likelihood of the vehicle pulling in front of you. This is called fuzzy input. However, some of your "input" is reasonably precise and non-fuzzy such as your speedometer reading. A human processing of all this information is not very precisely definable. This is called fuzzy processing. Fuzzy logic theorists would call it using fuzzy algorithms. (Algorithm is another word for procedure or program, as in a computer program).

Fuzzy logic is the way the human brain works, and we can mimic this in machines so they will perform somewhat like humans, not to be confused with Artificial Intelligence, where the so far unattainable goal is for machines to perform EXACTLY like humans.
BASIC DEFINITIONS

Some preliminary definitions are provided here and utilized throughout this course. You will see that data may not be changed or altered but the processing of such data might be. Additionally, data is used in multiple “groupings” or “categories” or “sets”. It is this multiple sets which gives fuzzy processing its flexibility.

Define CRISP DATA as precisely defined input. Examples might be: 5.2 inches, 8 feet, 45 mph, 15 / 8 (Cartesian coordinates on an X-Y axis). Each of these examples are defined as single data points; so 5.2 inches is not the same as 5.25 inches or greater than 5 inches or less than 6 inches.

Use the following crisp data for the following examples:

- Sally (female)
- Age: 14 (today is Sally’s birthday)
- Height: 4’ 5”
- Weight: 100 pounds
- Hair color: Dark Blonde
- Eye Color: Blue eyes

Fuzzy logic could tell that Sally is “not tall” compared to the entire population on earth or extremely tall compared to all members of her class in her current grade school. By defining set membership groups, we can categorize Sally as being part of every group with some portion of participation. That is, Sally can be described as a member of the group called “female giants” but the confidence level applied is extremely low (say 0.015). These confidence levels are based on the ‘degree of truth’ as follows:

- Crisp Facts – distinct boundaries
- Fuzzy Facts – imprecise boundaries
- Probability - incomplete facts

Example – Scout reporting an enemy:

- “Two to three tanks at grid NV 123456“ (Crisp)
- “A few tanks at grid NV 123456” (Fuzzy)
- “There might be 2 tanks at grid NV 54 (Probabilistic)

One further note, fuzzy analysis is not to be confused with reasoning or decision making under uncertainty. Crisp data, which can take on many forms, is required for the fuzzy processing to be applied. Uncertainty generally refers to a probability of realizing some future data which may be drawn from a known or unknown probability distribution. The probability of a “noon rain shower tomorrow providing 1 inch of rain” might be drawn from a Gaussian, Normal or other distribution defining this event of uncertainty. On another hand, given a range of crisp data about a potential rain tomorrow, fuzzy logic can be applied to decisions surrounding the crisp data event (i.e., after the fact).
BACKGROUND AND SOME HISTORY

The concept of Fuzzy Logic (FL) was conceived by Lotfi Zadeh, a professor at the University of California at Berkley. Fuzzy logic was developed in the 1960’s in order to provide mathematical rules and functions which permitted natural language queries. Fuzzy logic provides a means of calculating intermediate values between absolute true and absolute false with resulting values ranging between 0.0 and 1.0. With fuzzy logic, it is possible to calculate the degree to which an item is a member. For example, if a person is .83 of tallness, they are "rather tall." Fuzzy logic calculates the shades of gray between black/white and true/false.

In 1965 Dr. Zadeh published a paper "Fuzzy Sets" that formally developed multi-valued set theory and introduced the term fuzzy into technical literature.

E.H. Mamdani is credited with building the world's first fuzzy logic controller, after reading Dr. Zadeh's paper on the subject. Dr. Mamdani, London University, U.K., stated firmly and unequivocally that utilizing a fuzzy logic controller for speed control of a steam engine was much superior to controlling the engine by conventional mathematically based control systems and logic control hardware. Dr. Mamdani found that, using the conventional approach, extensive trial and error work was necessary to arrive at successful control for a specific speed set-point. Further, due to the non-linearity of the steam engine operating characteristics, as soon as the speed set-point was changed, the trial and error effort had to be done all over again to arrive at effective control. This did not occur with the fuzzy logic controller which adapted much better to changes, variations and non-linearity in the system.

Dr. Zadeh, as the principal founder of the fuzzy logic theory has earned numerous Awards, Fellowships and Honors and has contributed a large amount of research and publications to the field of knowledge representation. A brief summary of historical steps follows:

1965 Seminal Paper “Fuzzy Logic” by Prof. Lotfi Zadeh, Faculty in Electrical Engineering, U.C. Berkeley, Sets the Foundation of the “Fuzzy Set Theory”
1970 First Application of Fuzzy Logic in Control Engineering (Europe)
1975 Introduction of Fuzzy Logic in Japan
1980 Empirical Verification of Fuzzy Logic in Europe
1985 Broad Application of Fuzzy Logic in Japan
1990 Broad Application of Fuzzy Logic in Europe
1995 Broad Application of Fuzzy Logic in the U.S.
Interesting “historical” side notes

Before getting to the heart of fuzzy logic applied to basic control applications, it may be very informative to the reader to explain some of the nuances of “fuzz” and why many scientists and developers in OTHER countries embraced this faster than those based in the United States.

• The word “fuzzy” has an negative connotation in the US (Fuzzy thinking, fuzzballs, fuzzy haircuts, etc.); uphill battle with first impression
• “Fuzzy” phonetically translates to “smart” in Japanese

Not that the above ideas are the absolute reasons, but this has helped to contribute to the fact that Japan as well as the Asian and Pacific Rim countries have 80% of the patents of Fuzzy related inventions.

WHAT IS FUZZY ANYTHING AND WHY DO WE CARE

Artificial Intelligence

Artificial Intelligence (AI) is a form of computer reasoning designed to mimic that of the human reasoning process. Artificial intelligence may be thought of as the umbrella term for many forms of evolving technology (expert system, fuzzy logic, neural networks) that in some manner attempts to embody the computer with human-like capabilities (thinking, seeing, hearing, etc.) Fuzzy logic is a superset of conventional (Boolean) logic that has been extended to handle the concept of partial truth with truth values between "completely true" and "completely false".

Fuzzy logic is the attempt at formalization of approximate reasoning which is characteristic of the way in which humans reason in an environment of uncertainty and approximation. Fuzzy logic holds that all things are a matter of degree. Fuzzy logic has been used in applications areas such as project management, product pricing models, sales forecasting, criminal identification, process control and signal processing.

Fuzzy logic is used in system control and analysis design, because it shortens the time for engineering development and sometimes, in the case of highly complex systems, is the only way to solve the problem.

Fuzziness describes event ambiguity. It measures the degree to which an event occurs, not whether it occurs. Fuzzy sets have more flexible membership requirements that allow for partial membership in a set. The degree to which an object is a member of a fuzzy set can be any value from 0 to 1, rather than strictly 0 or 1 as in a traditional set. Structured properly, a fuzzy set allows a gradual transition from membership to non-membership.
For example, a man 6 feet tall may be a member of the fuzzy set TALL to degree 0.5. A man 5 feet 6 inches tall is TALL to degree 0.25, and a man 6 feet 6 inches tall is TALL to degree 0.75. In this example, the membership degrees (0.25, 0.50 and 0.75) are assigned by the developer; therefore, there is great flexibility in setting up the fuzzy logic system.

Fuzzy sets, which allow partial membership, provide a way for a computer to deal with this ambiguity by classifying the man as partially SHORT and partially TALL at the same time.

**Fuzzy vs. Randomness vs. Probabilities**

Fuzzy logic is a logic of fuzziness and not a logic which is itself fuzzy. This is analogous to probability. Probability laws are not random, just as laws of fuzziness are not vague.

Randomness describes the uncertainty of event occurrence. Whether an event occurs can be said to be "random". Generally, event occurrence can be measured, say, using past data which reports how often a particular event happened versus the range of all possible events. Prediction of weather events, such as rain or clouds, is based on historical outcomes in the past with identical or similar current inputs.

Fuzzy applications will come to understand that "partly cloudy" can mean a degree of sunshine. The probability of a cloudy day is definitive. The degree to which it is cloudy is fuzzy. The degree to which an event occurs can be said to be fuzzy. And to make a more blurred transition between randomness and fuzziness, probability is often applied to fuzzy events: satisfied customers, slight delay, partly cloudy.

**Fuzzy Sets, Fuzzy Logic and Fuzzy Operators**

In a conventional set theory, on which Boolean logic depends, a particular object is either a member of a given set of objects or it is not. This can be summarized with “crisp” IF statements: IF this, then that; or Not This, then the other.

An object in a fuzzy set has a degree of membership in a given set of objects that may be anywhere in the range 0 (completely not in the set) to 1 (completely in the set). For example, the set of TALL men could be defined to be all men 6 feet tall or taller. We would conclude a man 6 feet tall is TALL, but a man 5 feet 11 inches is not TALL.

In a traditional expert system, the rules either fire completely or not at all. This is because a traditional expert system uses “yes/no,” “black and white” logic to evaluate the premise of each rule; the application of a true Boolean logic system. In contrast, the rules in a fuzzy expert system fire to different degrees. Rather than an all-or-nothing response, the fuzzy rules produce “shades of gray” responses. In addition, more than one rule may fire for a given group of inputs, so the output of the expert system may be the combined result of several rules.
Fuzzy Logic

Fuzzy operators allow one to combine differing inputs into premises or rules/applications. The fuzzy logic operators are AND, OR and NOT. For an example, assume we operate a ‘Big and Tall’ gentlemen’s clothing store and wish to market to our target market. What is that market? We can say it is all “tall men”. But, what “height” do we say: men above “X” are in our market but below “X” they are not? If we define “X” as 6’-3”, we may be excluding a large population who feel they are tall at 6’-2”. We can apply fuzzy logic works through the use of fuzzy sets so that we include larger population groups in our marketing attempts.

Fuzzification of Data

An example of crisp data being converted into various fuzzy sets is fairly easy to understand. The following is an example of board lengths in a lumber yard:

- Very long boards: 12’ to 18’
- Long boards: 10’ to 15’
- Almost long boards: 8’ to 12.5’

A board with a length of 12’ will be a member of the three sets defined above. Note the value of set membership is not provided in this example.

Even before the creation of fuzzy logic, “fuzzification” of crisp data was evolving. Below is an example of how descriptive words can be converted to crisp data. Also, these charts can be used in reverse.

<table>
<thead>
<tr>
<th>Simpson 1944 Median</th>
<th>Hakel 1968 Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Always</td>
<td>99</td>
</tr>
<tr>
<td>Very Often</td>
<td>88</td>
</tr>
<tr>
<td>Usually</td>
<td>85</td>
</tr>
<tr>
<td>Often</td>
<td>78</td>
</tr>
<tr>
<td>Generally</td>
<td>78</td>
</tr>
<tr>
<td>Frequently</td>
<td>73</td>
</tr>
<tr>
<td>Rather Often</td>
<td>65</td>
</tr>
<tr>
<td>About as Often as not</td>
<td>50</td>
</tr>
<tr>
<td>Now and Then</td>
<td>20</td>
</tr>
<tr>
<td>Sometimes</td>
<td>20</td>
</tr>
<tr>
<td>Occasionally</td>
<td>20</td>
</tr>
<tr>
<td>Once in a While</td>
<td>15</td>
</tr>
<tr>
<td>Not Often</td>
<td>13</td>
</tr>
<tr>
<td>Usually not</td>
<td>10</td>
</tr>
<tr>
<td>Seldom</td>
<td>10</td>
</tr>
<tr>
<td>Hardly Ever</td>
<td>7</td>
</tr>
<tr>
<td>Very Seldom</td>
<td>6</td>
</tr>
<tr>
<td>Rarely</td>
<td>5</td>
</tr>
<tr>
<td>Almost Never</td>
<td>3</td>
</tr>
<tr>
<td>Never</td>
<td>0</td>
</tr>
</tbody>
</table>
The ideas of the charts created by Simpson and by Hakel imply that any data can be converted into fuzzy data.

**Defuzzification**

All fuzzy logic inference methods result in fuzzy values for all output information. In order to generate a single crisp output value in those cases that require it, a method is needed to pick a value that best represents the membership function; a way of defuzzifying a fuzzy set. For example, an inside temperature of 78 °F might be set as a “crisp” rule where the air condition comes on or not, depending on the actual temperature at the thermostat. However, a ‘feeling’ of “I am warm” or “I am hot” does not translate into a command for the thermostat.

Ranges can be set to define “warm”, “very warm”, “hot”, and so on. Crisp data can be converted to fuzzy data. Fuzzy logic can be applied to the data sets, thereby resulting in the data sets creating a definite decision. It is the last phase (the decision) we call defuzzification because this step takes the fuzzy sets and converts that analysis into a decision; a ‘crisp’ decision. Real world systems require a crisp, numerical result. A fuzzy system takes the combined fuzzy output and converts it into a crisp, numerical result through a process called defuzzification.

**COMPLEX EXAMPLE**

If you think about it, much of the information you take in is not very precisely defined, such as evaluation of the behavior of a vehicle entering from a side street and the likelihood of the vehicle pulling in front of you. We call this fuzzy input. However, some of your "input" is reasonably precise and non-fuzzy such as the speedometer reading. Your processing of all this information is not precisely definable. We call this fuzzy processing. Fuzzy logic theorists would call it using fuzzy algorithms (algorithm is another word for procedure or program, as in a computer program).

Suppose you are driving down a typical, two-way, six-lane street in a large city, one mile between signal lights. The speed limit is posted at 45 Mph. It is usually optimum and safest to "drive with the traffic," which will usually be going about 48 Mph. There will be some drivers weaving in and out and going more than 48 Mph and a few drivers driving exactly the posted 45 Mph. But, most drivers will be driving 48 Mph.

How do you define with specific, precise instructions "driving with the traffic?" It is difficult. But, it is the kind of processing humans do every day and do well. All drivers (yes, the good ones and the bad ones!) process these inputs by exercising "fuzzy logic" - receiving a large number of fuzzy inputs, somehow evaluating all the inputs in their human brains and summarizing, weighting and averaging all these inputs to yield an optimum output decision.
Fuzzy Logic

Think about it! Inputs being evaluated may include several images and considerations such as:

- Any "old clunkers" going real slow
- Any trucks holding up one of the lanes
- Side traffic entering from side streets
- How many cars are in front
- How fast are they driving
- Do the police ever set up radar surveillance on this stretch of road
- How much leeway do the police allow over the 45 Mph limit
- What do you see in the rear view mirror

Examples of setting up fuzzy sets

In looking at height as an example, below are 5 individuals along with their “crisp” data points, which are exact height measurements. We can define “Tall” as being anything above 5’ 10”. This leads to two additional sets: one defined with crisp data and the other defined using fuzzy logic. Note, the participation number (%) for a given individual (i.e. Bob having a 0.40 participation number) is determined by the system creator.

**Define: Tall > 5 Ft 10 inches**

<table>
<thead>
<tr>
<th>Description</th>
<th>Height</th>
<th>Crisp Analysis</th>
<th>Fuzzy Set Membership</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jim</td>
<td>6’-6”</td>
<td>in</td>
<td>0.95</td>
</tr>
<tr>
<td>Jon</td>
<td>6’-2”</td>
<td>in</td>
<td>0.8</td>
</tr>
<tr>
<td>Tom</td>
<td>5’-11”</td>
<td>in</td>
<td>0.6</td>
</tr>
<tr>
<td>Bob</td>
<td>5’-9”</td>
<td>out</td>
<td>0.4</td>
</tr>
<tr>
<td>Bill</td>
<td>5’-6”</td>
<td>out</td>
<td>0.2</td>
</tr>
</tbody>
</table>

A second example provided below targets river lengths.

**Define: Long River > 1,000 Meters**

<table>
<thead>
<tr>
<th>Description</th>
<th>Height</th>
<th>Crisp Analysis</th>
<th>Fuzzy Set membership</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nile</td>
<td>4,180</td>
<td>in</td>
<td>1</td>
</tr>
<tr>
<td>Mississippi</td>
<td>2,348</td>
<td>in</td>
<td>0.82</td>
</tr>
<tr>
<td>Danube</td>
<td>1,766</td>
<td>in</td>
<td>0.75</td>
</tr>
<tr>
<td>Rhine</td>
<td>820</td>
<td>out</td>
<td>0.4</td>
</tr>
<tr>
<td>Hudson</td>
<td>306</td>
<td>out</td>
<td>0.15</td>
</tr>
</tbody>
</table>
Now, applying the idea of fuzzy sets, a marketing person might be very interested in the following example. You can see there may be some gray areas when a perspective customer might not fit hard and crisp data.

**Fact:**
- Jill Plays Tennis, twice per year
- Marie lives half yr in France; half in Brazil
- "Heap of Sand"
  - remove a grain at a time…….

**Question:**
- Is Jill a tennis player?
- Is Marie a resident of France?
- When does the "heap" not be a "heap"?
  - Pile?

By having fuzzy sets, a marketing program might target both main bodies of potential customers along with marginally potential customers.

**Examples of fuzzy sets using Graphics**

**Graphic Example 1**

Here is an example describing a set of young people using fuzzy sets. In general, young people range from the age of 0 to 20. But, if we use this strict interval to define young people, then a person on his 20th birthday is still young (still a member of the set). But on the day after his 20th birthday, this person is now old (not a member of the young set).

How can one remedy this?

By relaxing the boundary between the strict separation of young and old. This separation can easily be relaxed by considering the boundary between young and old as "fuzzy". The figure below graphically illustrates a fuzzy set of young and old people.

![Graphical illustration of fuzzy set](image)

Notice in the figure that people whose ages are $\geq$ zero and $\leq$ 20 are complete members of the young set (that is, they have a membership value of one). Also note that people whose ages are $> 20$ and $< 30$ are partial members of the young set. For example, a person who is 25 would be young to the degree of 0.5. Finally people whose ages are $\geq 30$ are non-members of the young set.
**Graphic Example 2**

Here is an application demonstrating fuzzy membership: Consider the colorwheel. Remember that there are three primary colors: Red, Yellow, and Blue. These colors, separately, represent crisp sets. For example, true red is a non-member of true blue and of true yellow; true blue is a non-member of true yellow and of true red; yellow is a non-member of true red and of true blue. There is a boundary between these primary colors.

But, as we also learned in elementary school, it is possible to mix these colors with varying amounts of the true colors resulting in different shades of non-true colors. For example, mixing true red with true blue in equal portions of each will result in violet with a membership degree of 0.5 in true red and 0.5 in true blue. Different amounts of true red and true blue will result in varied membership values for the violet. The different violets represent the fuzzy boundaries between true red and true blue!

In this figure, the red line is a fuzzy set. To negate this fuzzy set, subtract the membership value in the fuzzy set from one. For example, the membership value at 5 is one. In the negation, the membership value at 5 would be zero (1-1=0). Another example, if the membership value is 0.4. In the negation, the membership value would be 0.6 (1-0.4=0.6).

Put the mouse over the image to see the negation of the fuzzy set (blue curve).

**Intersection**

In this figure, the red and green lines are fuzzy sets. To find the intersection of these sets take the minimum of the two membership values at each point on the x-axis (see the formal definition above). For example, in the figure the red fuzzy set has a membership of ZERO when x = 4 and the green fuzzy set has a membership of ONE when x = 4. The intersection would have a membership value of ZERO when x = 4 because the minimum of zero and one is zero.

Put the mouse over the image to see the intersection of the fuzzy sets (blue curve).
**Fuzzy Logic**

**Union**

In this figure, the red and green lines are fuzzy sets. To find the union of these sets take the maximum of the two membership values at each point on the x-axis (see the formal definition above). For example, in the figure the red fuzzy set has a membership of ZERO when \( x = 4 \) and the green fuzzy set has a membership of ONE when \( x = 4 \). The union would have a membership value of ONE when \( x = 4 \) because the maximum of zero and one is one.

**Instance for Example**

- Distance could be considered small or perfect
- Delta could be stable or growing
- What acceleration?

The example above is the kind of thinking our brain does when driving a car and monitor other cars around us. We have a ‘feel’ for our distance to another car, but that data is not crisp. We may have a ‘feel’ for our acceleration or even relative acceleration between cars; again not crisp data. Last, we have a ‘feel’ for the difference between the two cars while neither knowing exactly how fast we are traveling nor knowing exact distances or road conditions (determining braking distances).

For the example above, some of the controlling languages might be given as follows:
Rules for controlling a car:
- Variables are distance to car in front and how fast it is changing, delta, and acceleration to apply
- Sets are:
  - Very small, small, perfect, big, very big - for distance
  - Shrinking fast, shrinking, stable, growing, growing fast for delta
  - Brake hard, slow down, none, speed up, floor it for acceleration
- Rules for every combination of distance and delta sets, defining an acceleration set
- Relevant rules are:
  - If distance is small and delta is growing, maintain speed
  - If distance is small and delta is stable, slow down
  - If distance is perfect and delta is growing, speed up
  - If distance is perfect and delta is stable, maintain speed
- For first rule, distance is small has 0.75 truth, and delta is growing has 0.3 truth
- Advantages:
  - Allows use of numbers while still writing “crisp” rules
  - Allows use of “fuzzy” concepts such as medium
  - Biggest impact is for control problems:
    - Help avoid discontinuities in behavior
    - In the example problem strict rules would give discontinuous acceleration

**Applying Fuzzy Operators in real world scenarios**

Fuzzy logic has been used in applications areas such as project management, product pricing models, sales forecasting, criminal identification, process control and signal processing. To create a personal computer based fuzzy logic control system, we:

1. Determine the inputs.

2. Describe the cause and effect action of the system with "fuzzy rules" stated in plain English words.

3. Write a computer program to act on the inputs and determine the output, considering each input separately. The rules become "If-Then" statements in the program. (As will be seen below, where feedback loop control is involved, use of graphical triangles can help visualize and compute this input-output action.)

4. In the program, use a weighted average to merge the various actions called for by the individual inputs into one crisp output acting on the controlled system. (In the event there is only one output, then merging is not necessary, only scaling the output is needed.)
Fuzzy Logic

The fuzzy logic approach makes it easier to conceptualize and implement control systems. The process is reduced to a set of visualized steps. This is a very important point. Actually implementing a control system, even a simple control system, is more difficult than it appears. Unexpected aberrations and physical anomalies inevitably occur. Getting the processes working correctly ends up being a "cut and try" effort.

The fuzzy logic analysis and control method is, therefore:

1. Receiving one measurement or a large number of measurements, or other assessment of conditions existing in some system we wish to analyze or control.

2. Processing all these inputs according to human based, fuzzy "If-Then" rules, which can be expressed in plain language words.

3. Averaging and weighting the resulting outputs from all the individual rules into one single output decision or signal which decides what to do or tells a controlled system what to do. The output signal eventually arrived at is a precise appearing, defuzzified, "crisp" value. Refer to the following Fuzzy Logic Control/Analysis Method diagram:

The Fuzzy Logic Control-Analysis Method

Other areas to explore outside the scope of this presentation

Fuzzy Mathematics

- Fuzzy Numbers – almost 5, or more than 50
- Fuzzy Geometry – Almost straight lines
- Fuzzy Algebra – Not quite a parabola
- Fuzzy Calculus
- Fuzzy Graphs – based on fuzzy points
Some brief listings of the application of fuzzy controls in the real world along with companies utilizing these applications are provided herein.

General Fuzzified Applications:

- Quality Assurance
- Error Diagnostics
- Control Theory
- Pattern Recognition
- Otis Elevators
- Vacuum Cleaners
- Hair Dryers
- Air Control in Soft Drink Production
- Noise Detection on Compact Disks
- Cranes
- Electric Razors
- Camcorders
- Television Sets
- Showers

SUMMARY

The idea of fuzzy sets and analysis is not a new item as it was developed in the 1960’s. Since that time, many applications have been commercialized in the control system arena. Fuzzy logic allows the ability to utilize crisp data in such a way that data points can be associated with different fuzzy sets. Then, these sets themselves can be analyzed to create some outcome; being a crisp decision.

Creating fuzzy sets allows for characteristics to a factor rather than mere numbers. All the while, these characteristics can be assigned set participation numbers. A quick example with both descriptive ‘sets’ and outcomes is:

- Different characteristics of players
  - Strength: strong, medium, weak
  - Aggressiveness: meek, medium, nasty
- Outcomes
  - If meek and attacked, run away fast
  - If medium and attacked, run away slowly
  - If nasty and strong and attacked, attack back
- Control of a vehicle
  - Should slow down when close to car in front
  - Should speed up when far behind car in front

The major result in applying fuzzy logic to control systems is that this approach provides smoother transitions. There are no sharp boundaries in the application stage even though
a fuzzy might have such boundaries. That is to say, if data points are increasing and nearing an end point of one fuzzy set, that same data point can be a member of another fuzzy set near that fuzzy set’s midpoint. It might be that this second fuzzy set now controls the outcome of the system.

Classical set theory requires an object or data point to be either in or not in the set. Likewise, controls based on this logic would have binary decisions; one or off. Fuzzy set allows for data to be described as not completely in or out of a given set; a 6-foot person is 80% tall. In other words, fuzzy set theory allows for an object to be in a set by matter of degree:

- 1.0 => in the set
- 0.0 => not in the set
- 0.0 < object < 1.0 => partially in the set

The idea of fuzzy logic applied to software, computers, controllers and many other applications should not be a deterrent in using such products in the market. Most applications of fuzzy logic are first adopted and adapted overseas with users in North America coming later into the game.