Hydraulic Jumps and Supercritical & Non-Uniform Open Channel Flow

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Credit: 2 PDH

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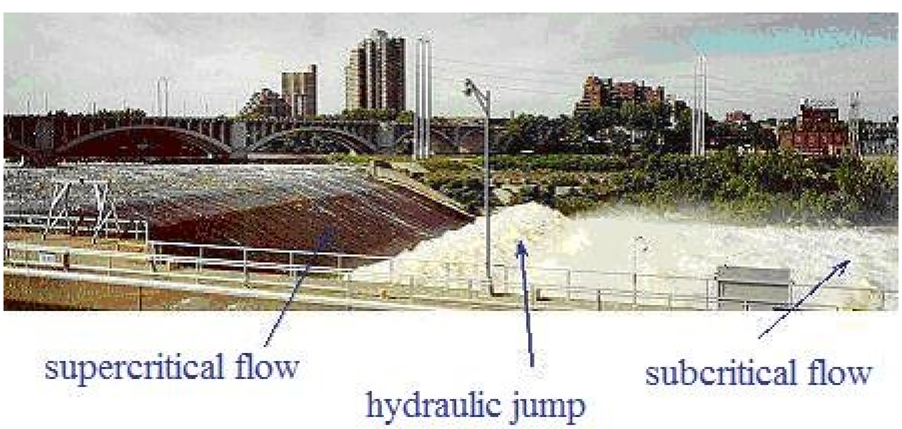
COURSE CONTENT

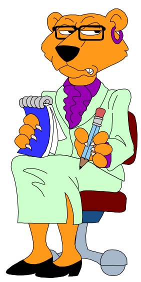
1. Introduction

Many examples of open channel flow can be approximated as uniform flow allowing the Manning equation to be used. Non-uniform flow calculations are needed, however, in some open channel flow situations, where the flow is clearly non-uniform. The concepts of supercritical, subcritical and critical flow, and calculations related to those three regimes of flow, are needed for non-uniform open channel flow analysis and calculations. Hence, in this course, the parameter called specific energy will first be used to introduce the concepts of critical, subcritical, and supercritical flow. Various calculations related to critical, subcritical and supercritical flow conditions will be presented, including hydraulic jump calculations. The thirteen possible types of gradually varied non-uniform flow surface profiles will then be presented and discussed. Also, the procedure and equations for step-wise calculation of gradually varied non-uniform surface profiles will be presented and illustrated with examples.



Hydraulic Jump at St. Anthony Falls
on the Mississippi River

2. Specific Energy as an Introduction to Supercritical, Subcritical and Critical Flow



Specifically, Just what is SPECIFIC ENERGY ?

The parameter, specific energy, can be used to help clarify the meaning of supercritical, subcritical and critical flow in an open channel. The definition of specific energy at any cross-section in an open channel is the sum of the kinetic energy per unit weight of the flowing liquid and the potential energy relative to the bottom of the channel. Thus an expression for specific energy is as follows:

$$E = y + V^2/2g \qquad (1)$$

Where: E is the specific energy in ft-lb/lb

y is the depth of flow above the bottom of the channel in ft

V is the average liquid velocity (= Q/A) in ft/sec

g is the acceleration due to gravity = 32.2 ft/sec^2

Another form of the equation with Q/A in place of V is:

$$E = y + Q^2/2A^2g \qquad (2)$$

The way that specific energy varies with depth of flow in an open channel can be illustrated by considering a rectangular open channel with bottom width b.

For such a channel, A = yb, where b is the channel width. Substituting for A in equation (2), gives:

$$E = y + Q^2/(2\,y^2b^2g) \qquad (3)$$

The parameter q, the flow rate per unit width of channel, is often used for a **rectangular channel**. The relationship between q and Q is thus: $q = Q/b$ or $Q = qb$. Substituting for Q in equation (3) gives:

$$E = y + q^2/(2\,y^2 g) \qquad (4)$$

Equation (4) can be used to prepare a plot of specific energy, E, as a function of the channel depth, y, for a selected value of q. Figure 1 below contains a table and graph showing how E varies with y for q = 15 cfs/ft. As shown in the graph, specific energy has high values for large values of y and it has high values for very low values of y. A close look at equation (4) provides an explanation. The first term in the equation (potential energy) makes E large as y becomes large. At very low values for y, the value of E is dominated by the second term in the equation (kinetic energy), which becomes large because of the small cross-sectional area of flow at small values for y.

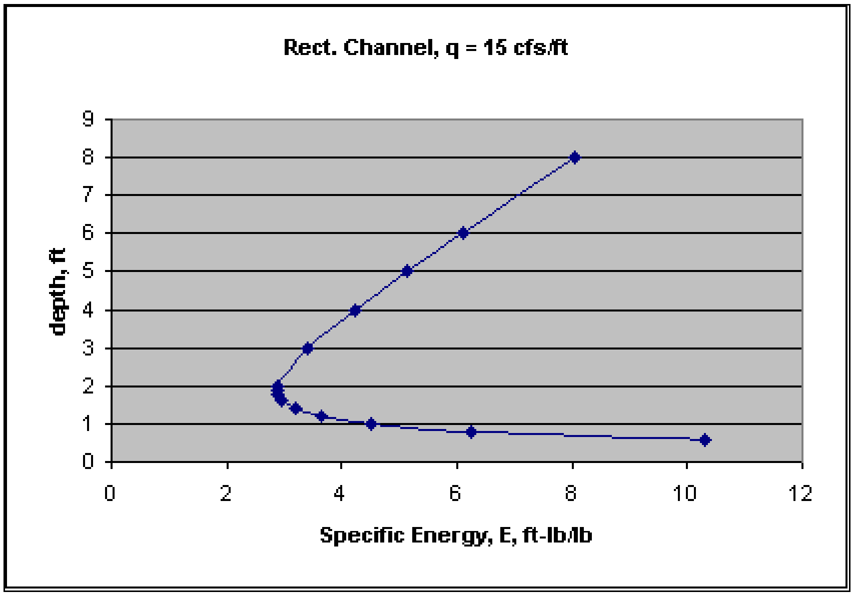
At some intermediate depth of flow, the specific energy must have a minimum value. The value of y at which the specific energy minimum occurs, is called the **critical depth** for the given value of q. From the table and graph below, it can be seen that the critical depth for q = 15 cfs/ft is 1.9 ft, accurate to 2 significant figures.

The symbol y_c is commonly used for critical depth and will be so used in this course. Through a little application of calculus, an equation for the critical depth, y_c, can be derived. The derivative of E with respect to y, dE/dy, must be determined from Equation (4), set equal to zero and solved for y. This will give an expression for y that gives either a minimum or maximum value for E. From inspection of the graph of E vs y in Figure 1, we can see that it must be a minimum value for E and that the value of y at that minimum is the critical depth, y_c. This procedure yields the following equation for y_c:

$$y_c = (q^2/g)^{1/3} \qquad (5)$$

Figure 1. Specific Energy vs Depth (Rect. channel, q = 15 cfs/ft)

E, ft-lb/lb	Y, ft
8.055	8
6.097	6
5.140	5
4.218	4
3.388	3
2.873	2
2.868	1.9
2.878	1.8
2.965	1.6
3.183	1.4
3.626	1.2
4.494	1
6.259	0.8
10.305	0.6



Example #1: Calculate the critical depth for a flow rate of 15 ft^3/sec in a rectangular open channel.

Solution: Using equation (5): $\mathbf{y_c} = (15^2/32.2)^{1/3} = (6.988)^{1/3}$ **= <u>1.912</u>**

Note that this value is consistent with the value of 1.9 from the Figure 1, but with more significant figures.

Any open channel flow having depth of flow less than critical depth ($y < y_c$) will be represented by a point on the lower leg of the graph above, and is called supercritical flow. Any open channel flow having depth of flow greater than critical depth ($y > y_c$) will be represented by a point on the upper leg of the graph above, and is called subcritical flow. The flow condition with $y = y_c$ is critical flow.

The Froude Number for Rectangular Channels

The Froude Number for flow in an open channel is defined as: **Fr = V^2/gy**, where V, y, and g are the average velocity, depth of flow, and acceleration due to gravity respectively. Fr, is a dimensionless parameter used in a variety of ways with open channel flow.

The equation below is obtained by substituting q = Q/b = VA/b = V(yb)/b = Vy, into equation (5) and simplifying.

$$V^2/gy_c = 1 \text{ or } Fr_c = 1$$

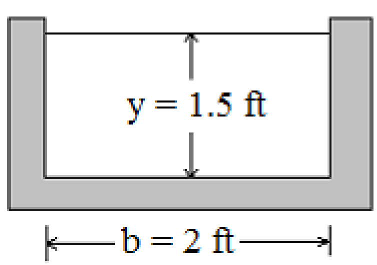
This equation shows that the Froude number is equal to one at critical flow conditions. Knowing that $y > y_c$ for subcritical flow, the Froude number must be less than 1 for subcritical flow. Similarly, since y < yc for supercritical flow, the Froude number must be greater than one for supercritical flow. Summarizing:

Fr < 1 for subcritical flow

Fr = 1 for critical flow

Fr > 1 for supercritical flow

Example #2: A rectangular open channel with bottom width = 2 ft, is carrying a flow rate of 12 cfs, with depth of flow = 1.5 ft. A cross-section of the channel is shown in the figure below. Is this subcritical or supercritical flow?



Solution: There is sufficient information to calculate the Froude Number, as follows:

$$A = by = (2)(1.5)\ ft^2 = 3\ ft^2$$

$V = Q/A = 12/3 = 4$ ft/sec

$Fr = V^2/gy = (4^2)/(32.2)(1.5) = 0.331$

Since Fr < 1, **this is subcritical flow**.

The Froude Number for Non-rectangular Channels

The definition for the Froude Number for flow in a channel with non-rectangular cross-section is $\mathbf{Fr = V^2/g(A/B)}$, where A is the cross-sectional area of flow and B is the surface width. A and B are shown in Figure 2, for a general, non-rectangular cross-section. Note that A/B = y for a rectangular channel, so the definition, $\mathbf{Fr = V^2/g(A/B)}$, reduces to V^2/gy for a rectangular channel. This is simply a more general definition for the Froude Number. The criteria noted above for the range of values of Fr for subcritical, supercritical and critical flow, apply to flow in non-rectangular channels as well.

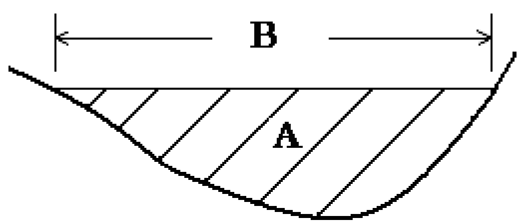
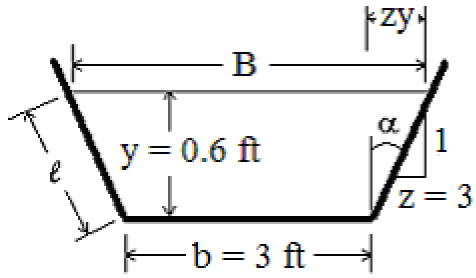


Figure 2. A and B for Non-rectangular Cross-section

Example #3: A trapezoidal open channel, with bottom width = 3 ft and side slope of horiz : vert = 3:1, is carrying a flow rate of 16 cfs, with depth of flow = 0.6 ft. Is this subcritical or supercritical flow? See the diagram below.



Solution: There is sufficient information to calculate the Froude Number, as follows: (recall from "Open Channel Hydraulics I", that $A = by + zy^2$).

$$A = by + zy^2 = (3)(0.6) + (3)(0.6^2)\ ft^2 = 2.88\ ft^2$$

$$V = Q/A = 16/2.88 = 5.55\ ft/sec$$

$$B = b + 2zy = 3 + (2)(3)(0.6) = 6.6\ ft$$

$$Fr = V^2/g(A/B) = (5.55^2)/(32.2)(2.88/6.6) = 2.19$$

Since Fr > 1, **<u>this is supercritical flow</u>**.

Calculation of Critical Slope

A channel with CRITICAL SLOPE will carry water at CRITICAL FLOW ? I guess that makes sense!

The slope that will give critical flow for a given flow rate in a channel of specified size, shape and Manning roughness is called the critical slope (S_c). The critical slope can be calculated from the Manning equation with parameters for critical flow conditions as follows:

$$\mathbf{Q = (1.49/n)A_c(R_{hc}^{2/3})S_c^{1/2}} \qquad (6)$$

Q and n will be known, along with channel shape and size parameters; A_c & R_{hc} will be functions of y_c ; and S_c is to be calculated from the equation. The critical depth, y_c, must be calculated first in order to get values for for A_c & R_{hc} . Two examples will be done to illustrate this type of calculation, the first with a rectangular channel and the second with a triangular channel.

Example #4: Find the critical slope for a rectangular channel with bottom width of 3 ft, Manning roughness of 0.011, carrying a flow rate of 16 cfs.

Solution: First calculate the critical depth from: $y_c = (q^2/g)^{1/3}$

Substituting values: $y_c = ((16/3)^2/32.2)^{1/3} = 0.9595$ ft

$A_c = by_c = (3)(0.9595) = 2.8785 \text{ ft}^2$

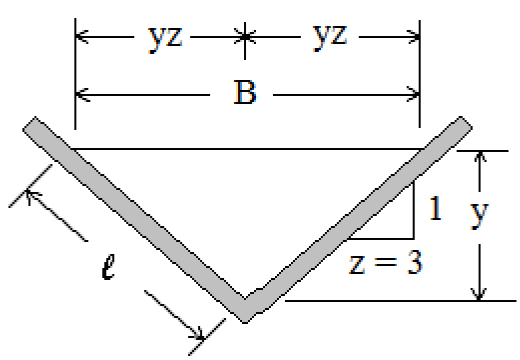
$P_c = b + 2y_c = 3 + (2)(0.9595) = 4.919$ ft

$R_{hc} = A_c/P_c = 2.8785/4.919 \text{ ft} = 0.5852$ ft

Substituting values into Eqn (6): $16 = (1.49/0.011)(2.8785)(0.5852^{2/3})S_c^{1/2}$

Solving for S_c gives: **$S_c = 0.00344$**

Example #5: Find the critical slope for a triangular channel with side slopes of horiz : vert = 3:1, Manning roughness of 0.012, carrying a flow rate of 12 cfs.



Solution: The critical depth (measured from the triangle vertex) can be calculated from the criterion that $Fr = V^2/g(A/B) = 1$ for critical flow, or

$$Fr_c = V_c^2/g(A_c/B_c) = 1.$$

From the figure above: $B_c = 2y_c z = 6y_c$

$$A_c = y_c^2 z = 3y_c^2$$

$$V_c = Q/A_c = 12/(3y_c^2)$$

Substituting expressions for V_c ,A_c , & B_c into the equation for Fr_c and setting it equal to 1 gives:

$$Fr_c = \frac{(12/3y_c^2)^2}{32.2(3y_c^2/6y_c)} = 1$$

Believe it or not, this simplifies to $0.99378/ y_c^5 = 1$

Solving: $y_c = 0.9987$ ft

Now, proceeding as in **Example #4:**

$$A_c = 3y_c^2 = (3)(0.9987)^2 = 2.992 \text{ ft}^2$$

$$P_c = 2[y_c^2(1 + z^2)]^{1/2} = 2[(0.9987^2)(1 + 3^2)]^{1/2} = 6.317 \text{ ft}$$

$$R_{hc} = A_c/P_c = 2.992/6.317 \text{ ft} = 0.4736 \text{ ft}$$

Substituting values into Eqn (6): $12 = (1.49/0.012)(2.992)(0.4736^{2/3})S_c^{1/2}$

Solving for S_c gives: **$\underline{S_c = 0.01530}$**

TERMINOLOGY: A bottom slope less than the critical slope for a given channel is called a **mild slope** and a slope greater than critical is called a **steep slope**.

3. The Hydraulic Jump

There are situations in which a subcritical slope will have supercritical flow taking place on it, such as flow under a sluice gate or a channel bottom slope changing from steep to mild. When this happens, the flow must slow down to the subcritical flow that can be maintained on the mild (subcritical) slope. There can be no gradual transition from supercritical to subcritical flow. The transition from supercritical to subcritical flow will always be an abrupt transition. That abrupt transition is called a hydraulic jump. Figure 2 and Figure 3 show a couple of physical situations that give rise to a hydraulic jump. The hydraulic jump is sometimes called rapidly varied flow.

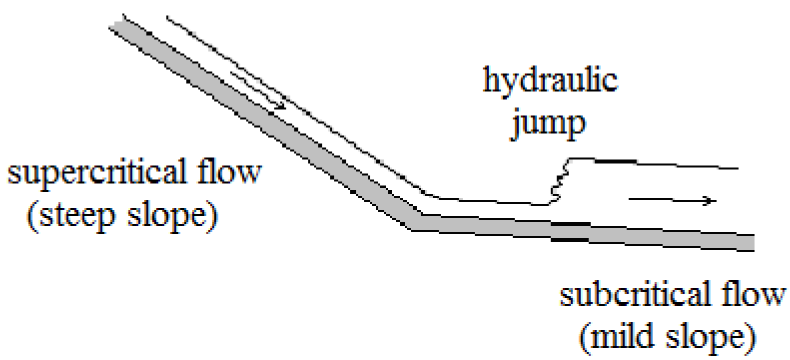


Figure 2. Hydraulic Jump due to slope change from Steep to Mild

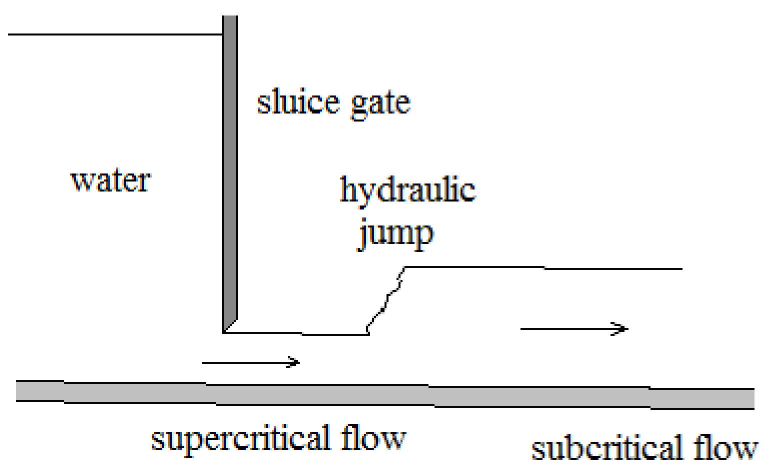


Figure 3. Hydraulic Jump following Flow Under a Sluice Gate

Figure 5 shows supercritical depth and velocity upstream of the jump and the subcritical depth and velocity downstream of the jump. These parameters are often used in hydraulic jump calculations. The supercritical (upstream) velocity and depth are represented by the symbols, V_1 and y_1. The subcritical (downstream) velocity and depth are represented by the symbols, V_2 and y_2.

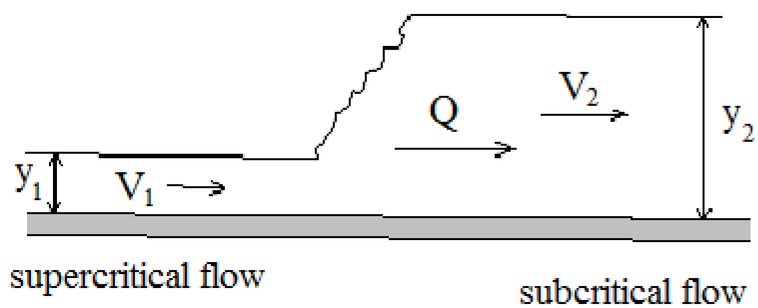


Figure 5. Hydraulic Jump with Upstream and Downstream Parameters

Equation (7) below can be derived by starting with the continuity equation, the energy equation, and the momentum equation, each written across the hydraulic jump. It gives a relationship among the depths upstream and downstream of the hydraulic jump and the Froude number upstream of the jump.

$$y_2/y_1 = (1/2)[\ -1 + (1 + 8Fr_1^2)^{1/2}\ ] \qquad (7)$$

Where: $Fr_1 = V_1^2/gy_1$

Example #6: The flow rate under a sluice gate in a 8 ft wide rectangular channel is 40 cfs, with a 0.7 ft depth of flow. If the channel slope is mild, will there be a hydraulic jump downstream of the sluice gate?

Solution: From the problem statement: $y = 0.7$ ft and $Q = 40$ cfs. Average velocity, V, can be calculated and then Fr can be calculated to determine whether this is subcritical or supercritical flow.

$$V = Q/A = Q/yb = 40/(0.7)(8) = 7.143 \text{ ft/sec}$$

$$Fr = V^2/gy = (7.143)^2/(32.2)(0.7) = 2.264 \quad \mathbf{(Fr > 1)}$$

Fr > 1, so the flow after the sluice gate is supercritical. The channel slope is mild, so there will be a hydraulic jump to make the transition from supercritical to subcritical flow.

Example #7: What will be the depth of flow and average velocity in the subcritical flow following the hydraulic jump of **Example #6?**

Solution: Equation (7) can be used with $Fr_1 = 2.264$ and $y_1 = 0.7$. Equation (7) becomes:

$$y_2/0.7 = (1/2)[\ -1 + (1 + 8(2.264)^2)^{1/2}\ ] = 3.702$$

$$\underline{\mathbf{y_2 = 2.591 \text{ ft}}}$$

$$V_2 = Q/A_2 = 40/(8)(2.591) = \underline{\mathbf{1.93 \text{ ft/sec} = V_2}}$$

4. Gradually Varied Open Channel Flow

Non-uniform flow with a smooth, gradual change in depth is called gradually varied flow. In contrast, rapidly varied flow refers to the flow in, before and after a hydraulic jump, where there is an abrupt transition from supercritical flow to subcritical flow.

Gradually Varied Flow Surface Profile Classifications

The thirteen possible gradually varied flow surface profiles are shown in Figure 6 on the next page. Also shown is the classification scheme, which makes use of the following five possible types of channel bottom slope:

i) mild (M): $(S < S_c)$,

ii) Steep (S): $(S > S_c)$,

iii) critical (C): $(S = S_c)$,

iv) horizontal (H): $(S = 0)$, and

v) adverse (A): (upward slope).

The three categories shown below are typically used as a classification of the relative values of actual depth of flow, y, normal depth, y_o, and critical depth, y_c.

i) category 1: $y > y_c$ & $y > y_o$;

ii) category 2: y is between y_c & y_o ;

iii) category 3: $y < y_c$ & $y < y_o$.

For example, a non-uniform surface profile on a mild slope with $y < y_c$ and $y < y_o$ is called an M_3 profile, and a non-uniform surface profile on a steep slope with $y > y_c$ and $y > y_o$ is called an S_1 profile. The entire classification and the terminology typically used is shown in Figure 6, on the next page.

Note that there cannot be an H_1 or A_1 surface profile, because neither a horizontal nor adverse slope has a normal depth. Neither of them can sustain uniform flow.

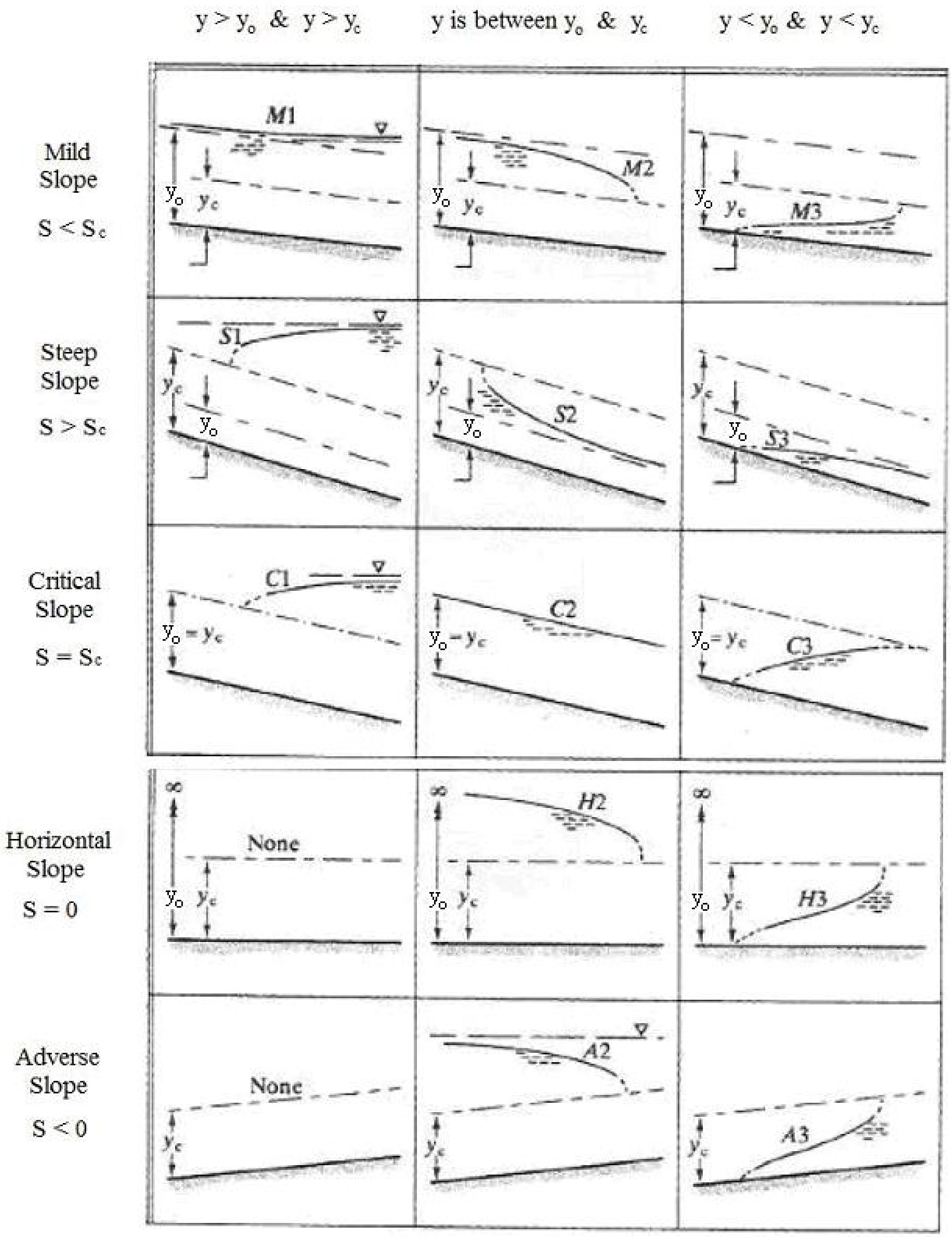


Figure 6. Gradually Varied Flow Surface Profile Classification

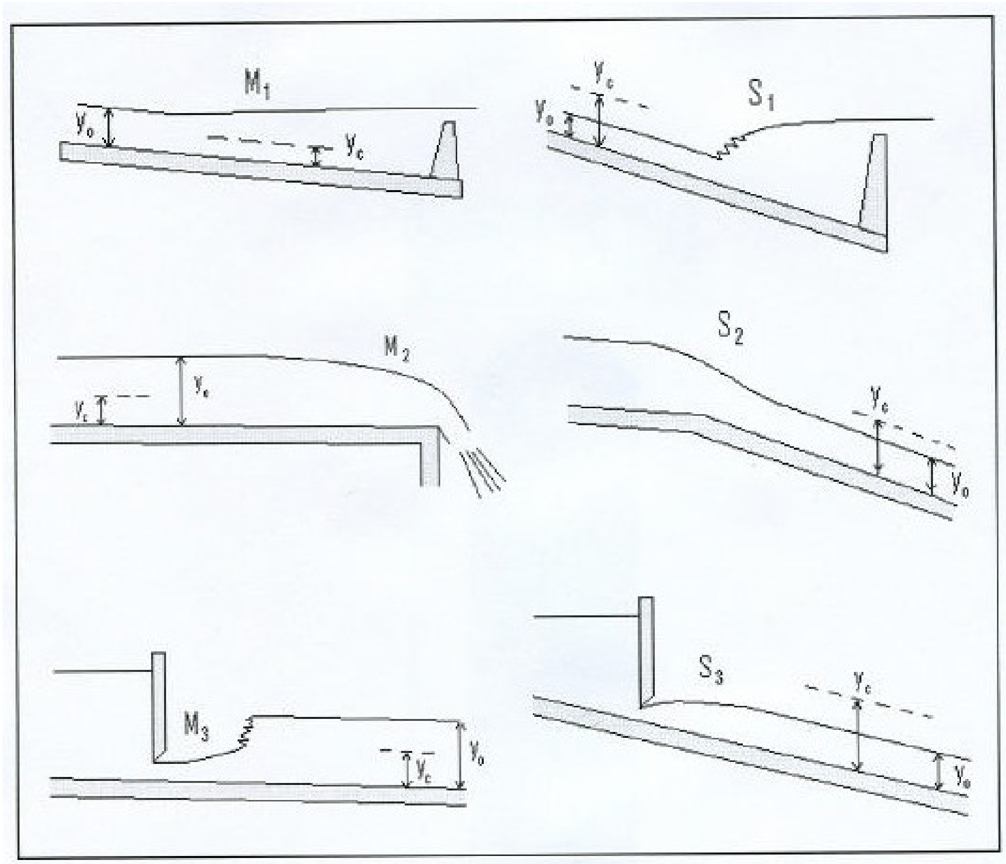


Figure 7. Examples of Mild and Steep Gradually Varied Surface Profiles

Figure 7 shows physical situations which lead to the six classifications of gradually varied flow surface profiles that are possible for mild and steep slopes.

Gradually Varied Flow Surface Profiles by Stepwise Calculations

In uniform open channel flow, the flow depth is constant, so the slope of the liquid surface (the surface slope) is the same as the channel bottom slope. In gradually varied open channel flow, however, the depth of flow is not constant, so the surface slope is different than the channel bottom slope. What's more, the surface slope isn't even constant for gradually varied flow.

If the depth of flow is increasing in the direction of flow, then the surface slope will be greater than the bottom slope. If the depth of flow is decreasing in the direction of flow, then the surface slope will be less than the bottom slope.

Figure 8 illustrates the general relationship between channel bottom slope and surface slope for uniform flow and for gradually varied non-uniform flow.

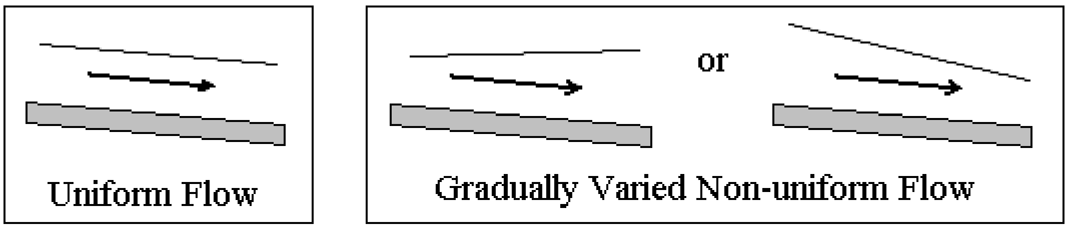


Figure 8. Uniform Flow and Gradually Varied Non-uniform Flow

Figure 9 shows a longitudinal section of a reach of open channel that has gradually varied non-uniform flow in it. This diagram will be used to help in developing an equation that can be used for stepwise calculation of a gradually varied flow surface profile (depth of flow vs length in direction of flow). The kinetic energy head and potential energy head at the inlet and outlet ends of the channel reach are shown in the diagram, along with the channel bottom slope and the head loss over the channel reach. The parameters in the diagram are summarized here:

Potential energy head of flowing water in (ft-lb/lb) = $y_1 + S_oL$

Kinetic energy head of flowing water in (ft-lb/lb) = $V_1^2/2g$

Potential energy head of flowing water out (ft-lb/lb) = y_2

Kinetic energy head of flowing water out (ft-lb/lb) = $V_2^2/2g$

Frictional head loss over channel reach of length L = h_L

Channel bottom slope = S_o

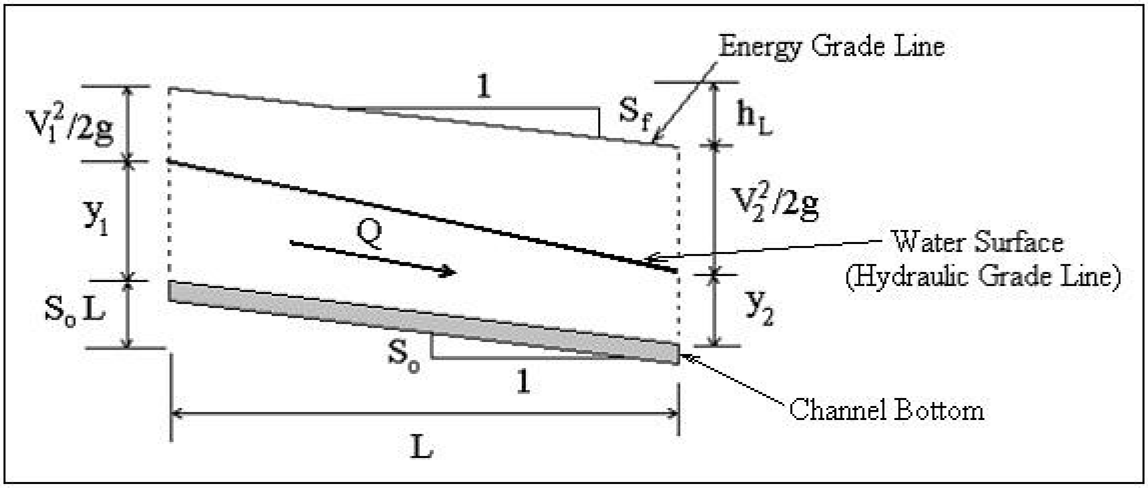


Figure 9. Gradually Varied, Non-uniform Flow in a Reach of Open Channel

A statement of the First Law of Thermodynamics (conservation of energy), as applied to the reach of channel in Figure 9, is:

Energy per lb of water flowing into the reach = Energy per lb of water flowing out of the reach + Frictional head loss over the reach of channel

Putting in the parameters from above, the equation becomes:

$$y_1 + S_oL + V_1^2/2g = y_2 + V_2^2/2g + h_L \qquad (8)$$

Note that the frictional head loss over the channel reach is equal to S_fL, where S_f is the slope of the energy grade line, or:

$$h_L = S_fL$$

Substituting into equation (8) and solving for $y_1 - y_2$, gives the following:

$$y_1 - y_2 = (V_2^2 - V_1^2)/2g + (S_f - S_o)L \qquad (9)$$

Equation (9) works very well for making a stepwise calculation of the gradually varied surface profile for a length of open channel. Each step can be taken to be a reach of channel such as that shown in Figure 9.

Equation (9) is typically used to calculate the length of channel, L, required for the depth of flow to change from y_1 to y_2. Parameters that must be known are the channel bottom slope, S_o, the size and shape of the channel cross-section, the Manning roughness of the channel surface, n, and the flow rate through the channel, Q.

Parameters in equation (9) that need to be calculated before calculating L are V_1, V_2, and S_f. With known channel flow rate, Q, and known channel shape and size, the velocity at each end of the channel reach can be calculated from:

$$V_1 = Q/A_1 \quad \text{and} \quad V_2 = Q/A_2$$

The value for S_f can be estimated from the Manning Equation:

$$S_f = [nQ/(1.49A_mR_{hm}^{2/3})]^2$$

NOTE: The slope used in the above equation is the slope of the energy grade line, not the bottom slope, as you are probably accustomed to using in the Manning equation. The slope to be used in the Manning equation is actually the slope of the energy grade line, but for uniform flow, the water surface slope, the energy grade line slope and the bottom slope are all the same, so the channel bottom slope can be used.

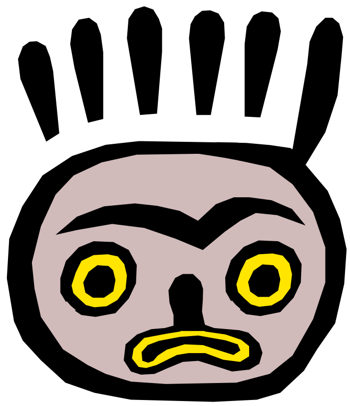
A_m and R_{hm} are mean values across the reach of channel, that is:

$$A_m = (A_1 + A_2)/2 \quad \text{and} \quad R_{hm} = (R_{h1} + R_{h2})/2$$

Some typical values for Manning roughness coefficient, n, for a variety of channel surfaces are given in Table 1, below.

Table 1. Manning Roughness Coefficient, n, for Selected Surfaces

Channel Surface	Manning Roughness Coefficient, n
Asbestos cement	0.011
Brass	0.011
Brick	0.015
Cast-iron, new	0.012
Concrete, steel forms	0.011
Concrete, wooden forms	0.015
Concrete, centrifugally spun	0.013
Copper	0.011
Corrugated metal	0.022
Galvanized Iron	0.016
Lead	0.011
Plastic	0.009
Steel - Coal-tar enamel	0.01
Steel - New unlined	0.011
Steel - Riveted	0.019
Wood stave	0.012



Watch out here ! This isn't hard, but there are a lot of steps and it can get tedious !!

Example #8: A rectangular wood stave flume ($n = 0.012$) is 4 ft wide and carries 40 cfs of water. The bed slope is 0.0005, and at a certain section the depth is 2.6 ft. Find the distance to the section where the depth is 2.4 ft.

Solution: A single step calculation will be used for the 0.2 ft change in depth. From the problem statement: $Q = 40$ cfs, $n = 0.012$, $S_o = 0.0005$, $b = 4$ ft, $y_1 = 2.6$ ft, and $y_2 = 2.4$ ft.

The other necessary parameters are calculated as follows:

$$V_1 = Q/A_1 = 40\ \text{ft}^3/\text{sec}/(4)(2.6)\text{ft}^2 = 3.846\ \text{ft/sec}$$

$$V_2 = Q/A_2 = 40\ \text{ft}^3/\text{sec}/(4)(2.4)\text{ft}^2 = 4.167\ \text{ft/sec}$$

$$A_m = (A_1 + A_2)/2 = [(4)(2.6) + (4)(2.4)]/2 = 10\ \text{ft}^2$$

$$R_{h1} = A_1/P_1 = (4)(2.6)/[4 + (2)(2.6)] = 1.130\ \text{ft}$$

$$R_{h2} = A_2/P_2 = (4)(2.4)/[4 + (2)(2.4)] = 1.091\ \text{ft}$$

$$R_{hm} = (R_{h1} + R_{h2})/2 = (1.130 + 1.091)/2 = 1.110\ \text{ft}$$

$$S_f = [nQ/(1.49A_mR_{hm}^{2/3})]^2 = [(0.012)(40)/(1.49(10)(1.110^{2/3})]^2$$

$$= 0.0009030$$

Substituting all of these values into equation (9) gives:

$$2.6 - 2.4 = (4.167^2 - 3.846^2)/(2)(32.2) + (0.0009030 - 0.0005)L$$

Solving for L: **$L = 397$ ft**

Example #9: In **Example #8,** is the 2.8 ft depth upstream or downstream of the 2.4 ft depth?

Solution: Since L as calculated from equation (9) is positive, **the 2.8 ft depth is upstream of the 2.4 ft depth.**

NOTE: Equation (9) is written with L being positive for y_1 being the upstream depth and y_2 being the downstream depth. In the solution to **Example #8**, y_1 was set to be 2.6 ft and y_2 was set to be 2.4 ft (the flow was assumed to be from 2.6 ft to 2.4 ft of depth. Since L came out positive in the calculation, this confirmed that the assumed direction of flow was correct. If L had come out negative, it would have meant that the flow was in the opposite direction.

As you can see from the diagrams in Figure 7, the depth can be either increasing or decreasing in the direction of flow, depending upon which type of non-uniform, gradually varied flow is present. You cannot simply assume that the flow is from the higher to lower depth of flow.

Example #10: a) Which classification of non-uniform, gradually varied flow is the flow in Example #8?

b) What is a physical situation that would lead to the type of surface profile in Example #8?

Solution: a) Calculation of critical depth and critical slope for the channel will often provide enough information to determine the gradually varied flow classification. Let's try:

The critical depth is given by equation (5):

$$y_c = (q^2/g)^{1/3} = [(40/4)^2/32.2]^{1/3} = 1.46 \text{ ft}$$

The critical slope can be calculated from equation (6):

$$Q = (1.49/n)A_c(R_{hc}^{2/3})S_c^{1/2}$$

Substituting known values gives:

$$40 = (1.49/0.012)[(1.46)(4)][(1.46)(4)/(4 +(2)(1.46)]^{2/3}(S_c^{1/2})$$

Solving for S_c gives: $S_c = 0.00326$

The given channel bottom slope, S_o, is 0.0005, which is less than S_c, so this channel slope is mild. From Figure 6, one can see that M2 is the only gradually varied profile on a mild slope that has the depth of flow decreasing in the direction of flow, as in Example 8, so **the gradually varied surface profile in Example #8 must be M2**.

b) The diagrams in Figure 7 shows that **a dropoff is an example of a physical situation that would lead to an M2 surface profile.**

Example #11: A rectangular channel is 20 ft wide, has a slope of 0.0003 (which is a mild slope) and Manning roughness of 0.015. The normal depth for this channel is 10 ft when it is carrying its current flow of 1006 cfs. Due to an obstruction, the depth of flow at one point in the channel is 16 ft. Determine the length of channel required for the transition from the 16 ft depth back to a depth of 11 ft. Use step-wise calculations with depth increments of one foot.

Solution: The physical situation as described for this example is like that shown in Figure 7 for an M1 surface profile. That is the depth is increasing in the direction of flow in order to pass over an obstruction. A set of calculations like those of **Example #8** will be done five times (for 11 to 12 ft; for 12 to 13 ft; etc, up to 15 to 16 ft). For repetitive calculations like this it is convenient to use a spreadsheet such as Excel.

The table below is copied from the Excel spreadsheet in which the calculations were made. In each column, calculations are made as shown above for **Example #8.** It can be seen that the distance for the transition from a depth of 11 ft to a depth of 16 ft is **35,412 feet or 6.71 miles**. The positive sign for L shows that the flow is indeed from the 11 ft depth to the 16 ft depth, which confirms what we already knew from the problem statement and the understanding that this is an M1 surface profile.

Table 2. Excel Spreadsheet Solution to Example #11

Q, cfs	1006	1006	1006	1006	1006
y_1, ft	11	12	13	14	15
y_2, ft	12	13	14	15	16
A_1, ft^2	220	240	260	280	300
A_2, ft^2	240	260	280	300	320
A_m, ft^2	230	250	270	290	310
V_1, ft/s	4.573	4.192	3.869	3.593	3.353
V_2, ft/s	4.374	4.024	3.726	3.469	3.245
R_{h1}, ft	5.238	5.455	5.652	5.833	6.000
R_{h2}, ft	5.455	5.652	5.833	6.000	6.154
R_{hm}, ft	5.346	5.553	5.743	5.917	6.077
S_f	0.000207	0.000167	0.000137	0.000114	9.62E-05
DL, ft	11097	7672	6231	5448	4962
Cumul. L, ft	11097	18770	25001	30450	**35412**
Cumul. L, mi	2.10	3.55	4.74	5.77	**6.71**

Example #12: Prepare a plot of depth vs distance along the channel to show the shape of the surface profile for the flow described in Example #11.

Solution: This can be accomplished by plotting y vs cumulative L using values from Table 2. Figure 10 on the next page shows the plot, prepared with Excel. The table of values for L and y that were used for the plot is shown. The shape of the surface profile is the same as that for the M1 profile shown in Figure 7.

L, mi	y, ft
0	11
2.1	12
3.55	13
4.74	14
5.77	15
6.71	16

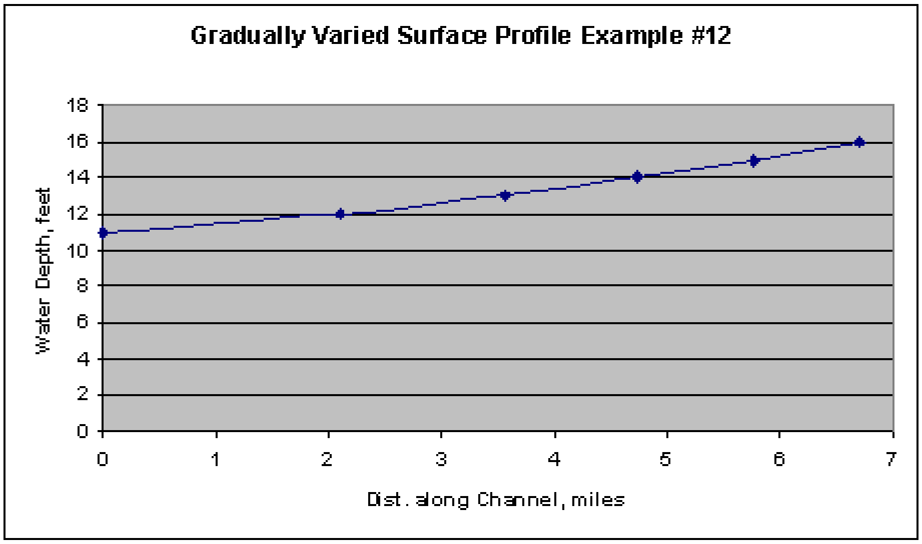


Figure 20. Surface Profile Plot for Example #12

5. Summary

The value of the Froude number can be used to determine whether a particular open channel flow is subcritical or supercritical flow. A hydraulic jump will make the abrupt transition from supercritical to subcritical flow, whenever supercritical flow is present on a mild slope that cannot maintain the supercritical velocity. Gradually varied non-uniform flow is any non-uniform flow in which the depth is changing gradually and smoothly instead of abruptly and turbulently as in a hydraulic jump. There are only 13 possible gradually varied flow variations. These 13 variations are typically classified, based on the slope of the channel and the relationships among y, y_c, & y_o. A particular gradually varied flow water surface profile can be calculated as depth versus distance along the channel using a step-wise calculation, which was discussed and illustrated with examples in this course.

6. Related Links and References

1. Munson, B. R., Young, D. F., & Okiishi, T. H., *Fundamentals of Fluid Mechanics,* 4th Ed., New York: John Wiley and Sons, Inc, 2002.

2. Chow, V. T., *Open Channel Hydraulics,* New York: McGraw-Hill, 1959.

Websites:

1. Indiana Department of Transportation Design Manual, available on the internet at: http://www.in.gov/dot/div/contracts/standards/dm/index.html.